

1 Activity Coefficients from Molecular Simulations using the OPAS Method

2 Maximilian Kohns,^{1, a)} Martin Horsch,¹ and Hans Hasse¹

3 *University of Kaiserslautern, Laboratory of Engineering Thermodynamics,*

4 *Erwin-Schrödinger Str. 44, D-67663 Kaiserslautern, Germany*

5 A method for determining activity coefficients by molecular dynamics simulations is
6 presented. It is an extension of the OPAS method (osmotic pressure for the activity of
7 the solvent) developed in previous work for studying the solvent activity in electrolyte
8 solutions. That method is extended here to study activities of all components in
9 mixtures of molecular species. As an example, activity coefficients in liquid mixtures
10 of water and methanol are calculated for 298.15 K and 323.15 K at 1 bar using
11 molecular models from the literature. These dense and strongly interacting mixtures
12 pose a significant challenge to existing methods for determining activity coefficients
13 by molecular simulation. It is shown that the new method yields accurate results
14 for the activity coefficients which are in agreement with results obtained with a
15 thermodynamic integration technique. As the partial molar volumes are needed in
16 the proposed method, the molar excess volume of the system water + methanol is
17 also investigated.

^{a)}Corresponding author; maximilian.kohns@mv.uni-kl.de; phone: +49-631 / 205-3216; fax: +49-631 / 205-3835.

18 I. INTRODUCTION

19 Chemical potentials or related properties like activities or activity coefficients are needed
20 for the determination of phase equilibria, and therefore their computation is a commonly en-
21 countered task in molecular simulation studies. Most methods for determining the chemical
22 potential in molecular simulations are based on particle insertion. Often, the test particle
23 method of Widom¹ is used. However, in dense systems, that method tends to fail because of
24 extremely high energies upon insertion of the 'ghost' particles. To overcome this difficulty,
25 different elaborate sampling schemes have been suggested, e.g. the gradual insertion or ther-
26 modynamic integration techniques^{2,3}. But still, for dense and strongly interacting systems,
27 long simulations are usually required, and method-specific simulation parameters have to be
28 determined by trial-and-error beforehand. Despite this, the methods may still turn out to
29 be unfeasible.

30 In this work, a new approach for obtaining activity coefficients in the liquid phase in molec-
31 ular dynamics simulations is presented. It builds on previous work of our group in which the
32 OPAS method (osmotic pressure for the activity of the solvent)^{4,5} was developed based on
33 ideas described by Murad and co-workers⁶⁻⁹. In its previous version^{4,5}, the OPAS method
34 was developed for determining the solvent activity in electrolyte solutions. In the present
35 work, it is extended for determining activities of all species in mixtures of molecular species,
36 similar to the approach of Crozier and Rowley¹⁰. To demonstrate the feasibility and accu-
37 racy of the new method, it is applied to determine activity coefficients in binary mixtures
38 of water and methanol at 298.15 K and 323.15 K and 1 bar.

39 Binary mixtures of water and methanol have been the subject of many experimental and
40 simulation studies. On the one hand, this is due to the importance of these substances

41 in many applications. On the other hand, as both water and methanol are strongly polar
42 and hydrogen bonding, their mixture has often been used as a test case for thermodynamic
43 models. The current version of the Dortmund Data Bank¹¹ lists more than 10,000 experi-
44 mental data points for this system. Especially vapor-liquid equilibria and excess properties
45 have been studied by many groups in a wide range of conditions. Mixtures of water and
46 methanol have also been studied extensively using molecular simulation (e.g. in Refs. ^{10,12-41}).
47 In most of these studies, the structure of the solution is investigated. Some publications also
48 address hydrogen bonding statistics^{12,13,18,28,29,38,41}, excess properties^{18-21,27,33,34,36,38-41} and
49 transport properties^{16,18,21,22,32,39,41}. In contrast, the number of molecular simulation stud-
50 ies of the vapor-liquid equilibrium or the related activity coefficients in the liquid phase is
51 small^{10,23,26,35}, due to the fact that these properties are difficult to compute accurately in
52 molecular simulations for dense liquid mixtures with strong interactions. **Activity coefficients**
53 **are derivatives of the free energy, so that they cannot be obtained by standard sampling**
54 **and instead, special algorithms have to be designed for their calculation. As outlined above,**
55 **most of the common algorithms to do so are based on particle insertions, which become very**
56 **cumbersome for systems of strongly interacting particles.**
57 In the present work, it is shown that the proposed extension of the OPAS method enables
58 an accurate calculation of activity coefficients, even in challenging mixtures such as water +
59 methanol. Since partial molar volumes are needed in the evaluation of the OPAS simulation
60 results, the molar excess volume of the mixture is also investigated. **To demonstrate the**
61 **viability of the extended OPAS approach, we compare its results to results obtained with a**
62 **thermodynamic integration technique.**
63 **The paper is structured as follows: In Section II, we briefly describe the employed molecular**
64 **models. The rest of Section II is devoted to the derivation and presentation of the exten-**

65 sion of the OPAS approach. This includes a procedure for obtaining partial molar volumes.
66 In Section III, these volumetric properties are discussed first. Then, the OPAS simulation
67 results are presented and compared to the results from thermodynamic integration, before
68 we conclude in Section IV.

69 II. MOLECULAR MODELS AND SIMULATION METHODS

70 A. Molecular Models

71 In the present study, the TIP4P/2005 water model⁴² and a methanol model from previous
72 work of our group⁴³ are used. Both are known to give good predictions of pure component
73 properties. Both models are rigid and non-polarizable and consist of Lennard-Jones (LJ)
74 sites and partial charges. The potential is therefore

$$\begin{aligned}
U &= U_{\text{LJ}} + U_{\text{CC}} \\
&= \sum_{i=1}^{N-1} \sum_{j=i+1}^N \left\{ \sum_{a=1}^{n_i^{\text{LJ}}} \sum_{b=1}^{n_j^{\text{LJ}}} 4\epsilon_{ijab} \left[\left(\frac{\sigma_{ijab}}{r_{ijab}} \right)^{12} - \left(\frac{\sigma_{ijab}}{r_{ijab}} \right)^6 \right] + \sum_{c=1}^{n_i^e} \sum_{d=1}^{n_j^e} \frac{1}{4\pi\epsilon_0} \frac{q_{ic}q_{jd}}{r_{ijcd}} \right\}. \quad (1)
\end{aligned}$$

75 Here, a , b , c and d denote sites, i and j denote molecules, ϵ_0 is the vacuum permittivity,
76 ϵ_{ijab} and σ_{ijab} are the Lennard-Jones energy and size parameters, r_{ijab} and r_{ijcd} are site-site
77 distances, and q_{ic} and q_{jd} are the magnitudes of the partial charges.

78 To describe the interaction between unlike LJ sites, the Lorentz-Berthelot combining rules
79 are used:

$$\sigma_{ijab} = \frac{\sigma_{iiaa} + \sigma_{jjbb}}{2}, \quad (2)$$

$$\epsilon_{ijab} = \sqrt{\epsilon_{iiaa}\epsilon_{jjbb}}. \quad (3)$$

80 B. Computation of Activity Coefficients with the OPAS Method

81 The OPAS method was developed in previous work with the aim of computing activities
82 in electrolyte solutions. It is only introduced briefly here, for details, the reader is referred
83 to our previous publications^{4,5}.

84 OPAS simulations are molecular dynamics simulations of the osmotic equilibrium between
85 a pure component phase (') and a mixture phase (") which are in contact via a virtual
86 semipermeable membrane. There is only one permeable component i . The membrane is
87 realized in the simulation by applying an external force field on all molecules but those of
88 component i to keep them inside the mixture phase ("), so that the phase (') contains pure i .
89 The osmotic pressure Π , which is the pressure difference between the two phases, is sampled
90 as the membrane force per area.

91 In OPAS simulations, the temperature T and the pressure p' of the pure component phase
92 are specified. The simulation results are the osmotic pressure Π and the composition of
93 the mixture phase \underline{x} . The goal is the computation of the activity coefficient $\gamma_i(T, p', \underline{x})$ of
94 the permeable component i at the specified temperature T and pressure p' as well as the
95 composition of the mixture phase \underline{x} .

96 In the following, it is shown how $\gamma_i(T, p', \underline{x})$ is obtained. Normalizing the chemical potentials
97 according to Raoult and rearranging the equilibrium condition for the osmotic equilibrium
98 described above yields

$$\mu_i^{\text{pure}}(T, p'') - \mu_i^{\text{pure}}(T, p') = -RT \ln a_i(T, p'', \underline{x}). \quad (4)$$

99 Herein, μ_i and a_i are the chemical potential and activity of the permeable component i , and
100 R is the universal gas constant. Combining Eq. (4) with

$$\mu_i(T, p', \underline{x}) = \mu_i^{\text{pure}}(T, p') + RT \ln a_i(T, p', \underline{x}) \quad (5)$$

$$\mu_i(T, p'', \underline{x}) = \mu_i^{\text{pure}}(T, p'') + RT \ln a_i(T, p'', \underline{x}) \quad (6)$$

yields

$$\mu_i(T, p'', \underline{x}) - \mu_i(T, p', \underline{x}) = -RT \ln a_i(T, p', \underline{x}). \quad (7)$$

Using $\Pi = p'' - p'$ this leads to

$$\int_{p'}^{p'+\Pi} v_i(T, p, \underline{x}) dp = -RT \ln a_i(T, p', \underline{x}). \quad (8)$$

Splitting the activity a_i into the mole fraction x_i and the activity coefficient γ_i finally yields the desired relation

$$\ln \gamma_i(T, p', \underline{x}) = -\frac{1}{RT} \int_{p'}^{p'+\Pi} v_i(T, p, \underline{x}) dp - \ln x_i. \quad (9)$$

101 In Eq. (9), T and p' are specified, and Π and x_i are OPAS simulation results. Additionally,
 102 the partial molar volume $v_i(T, p, \underline{x})$ is needed for the determination of the activity coefficient,
 103 and an integration over the pressure has to be carried out. In the present work, v_i was
 104 determined from the molar excess volume v^E as described in Section II C.

105 The relation between the activity and the osmotic pressure, cf. Eq. (9) has also been exploited
 106 in a recent paper by Smith et al.⁴⁴. They, however, proceed in the opposite direction: A
 107 relation similar to Eq. (9) is employed to calculate the osmotic pressure in an electrolyte
 108 solution from knowledge of the solvent activity.

109 To obtain the N activity coefficients in a mixture of N components, N independent OPAS
 110 simulation runs have to be carried out. In each run, a different component i is treated as
 111 the permeable component, while all others cannot permeate the membrane. By the present
 112 method, the mixture phase composition \underline{x} is not specified directly. Rather, the composition

113 is a simulation result. However, the initial conditions of the run can generally be chosen
 114 such that the final result for the composition is close to any desired value. The OPAS
 115 simulation runs are extremely simple, fast and robust compared to simulations employing
 116 particle insertion methods.

117 C. Computation of Molar Excess Volumes

The molar excess volume v^E of a mixture of N components is defined as:

$$v^E = v - \sum_{i=1}^N x_i v_i^{\text{pure}}, \quad (10)$$

where v is the molar volume of the mixture and v_i^{pure} are the pure component molar volumes. The molar excess volume is a function of temperature, pressure and composition: $v^E = v^E(T, p, \underline{x})$. In the present work, only binary mixtures of two components i and j are studied, so that Eq. (10) becomes

$$v^E = v - x_i v_i^{\text{pure}} - x_j v_j^{\text{pure}}. \quad (11)$$

118 The molar excess volume is computed here by performing a series of simulations at different
 119 compositions in the NpT ensemble and applying Eq. (11). As the osmotic pressures obtained
 120 in OPAS simulations can reach very high values, cf. Section III B, such simulations are carried
 121 out at several isobars up to elevated pressures. For each isobar, the molar excess volumes
 122 can be correlated by a Redlich-Kister type expression:

$$v^E = x_i x_j \left(A_0 + \sum_k A_k (x_i - x_j)^k \right), \quad k = 1, 2, \dots, \quad (12)$$

123 where A_0 and the A_k are fitting parameters. Adjusting Eq. (12) to the NpT simulation
 124 results is a standard polynomial regression. From that correlation, the partial molar volume
 125 v_i of component i can be obtained as

$$v_i = \left(\frac{\partial V}{\partial n_i} \right)_{T,p,n_{j \neq i}} = v_i^{\text{pure}} + v^{\text{E}} + x_j \left(\frac{\partial v^{\text{E}}}{\partial x_i} \right)_{T,p,n_{j \neq i}} \quad (13)$$

where

$$\begin{aligned} \left(\frac{\partial v^{\text{E}}}{\partial x_i} \right)_{T,p,n_{j \neq i}} &= (x_j - x_i) \left(A_0 + \sum_k A_k (x_i - x_j)^k \right) \\ &+ 2x_i x_j \left(\sum_k A_k k (x_i - x_j)^{(k-1)} \right), \quad k = 1, 2, \dots \end{aligned} \quad (14)$$

126 D. Details on the Simulations in the System Water + Methanol

127 In the present study, binary mixtures of water and methanol were considered. The aim
 128 was to determine activity coefficients at $p' = 1$ bar and either $T = 298.15$ K or $T = 323.15$ K
 129 over a wide concentration range. Details of the OPAS simulations and the standard NpT
 130 simulations for obtaining molar excess volumes are given in the Appendices A 1 and A 2,
 131 respectively. To validate the results for the activity coefficients determined with the OPAS
 132 method, they were also determined from independent simulations employing a thermody-
 133 namic integration technique^{3,45}. Technical details of these simulations are given in Appendix
 134 A 3.

135 III. RESULTS AND DISCUSSION

136 Simulation results of the NpT simulations for obtaining the molar excess volumes are
 137 given in Table I, and the OPAS simulation results are given in Table II. Table III con-
 138 tains the results of the simulations using thermodynamic integration to validate our OPAS
 139 computations. In all tables, numbers in parentheses indicate the simulation uncertainties
 140 in the last given digit. The uncertainties for v^{E} were determined from error propagation of
 141 the simulation results for ρ via Eq. (11). The uncertainties in the evaluation of γ_i from the

142 OPAS results were then estimated by using the simulation results for v^E disturbed by their
143 uncertainties in the correlation fitting, cf. Eq. (12), combined with error propagation of the
144 uncertainty of Π . The uncertainties in the evaluation of γ_i from the simulations employing
145 thermodynamic integration were estimated by error propagation of the simulation results
146 for $\tilde{\mu}_i$, cf. Eq. (A2) in Appendix A 3.

147 Since the focus of the present work is to demonstrate the viability of the OPAS method, we
148 discuss the comparison of the model predictions to experimental data only briefly.

149 A. Molar Excess Volumes and Partial Molar Volumes

150 First, the molar excess volumes of binary mixtures of water and methanol are discussed.
151 The molecular simulation results are compared to correlations of experimental data for the
152 two temperatures studied here in Fig. 1. The results of the predictive simulations with the
153 Lorentz-Berthelot rules capture the trends of the experimental data reasonably. The location
154 of the minimum in v^E at approximately equimolar composition is predicted correctly, how-
155 ever, its magnitude is underestimated by about 20%. The experimental data show that the
156 difference between the two temperatures is almost negligible, and the molecular simulation
157 results also capture this feature well. Quantitative agreement with the experimental data
158 can probably be achieved by introducing an adjustable parameter in the mixing rules^{20,27}.
159 In the present work, however, our main concern is demonstrating the viability of the OPAS
160 method for computing activity coefficients, and quantitative agreement with the experimen-
161 tal data is less important, so that we refrain from fitting binary interaction parameters.
162 Similar simulations were carried out at higher pressures. The simulation data were described
163 by Redlich-Kister correlations, cf. Eq. (12). The results are shown in Fig. 2, the fit param-
164 eters are reported in Table I. Up to pressures of about $p = 10$ MPa, the pressure effect on

165 the partial molar volumes is small, but it becomes increasingly important at the elevated
166 pressures. There, the partial molar volumes of both water and methanol exhibit a maxi-
167 mum. Comparing the two different temperatures, the curves are qualitatively similar. The
168 main difference is a vertical shift of the curves resulting from the temperature dependence
169 of the pure component molar volumes, which are in good agreement with the molar volumes
170 computed from equations of state for both studied temperatures, cf. Fig. 2. A more detailed
171 comparison of the partial molar volumes obtained from the simulations to experimental data
172 is presented in the Supplementary Material.

173 B. Osmotic Pressures and Activity Coefficients

174 The osmotic pressures obtained by OPAS simulations at the two studied temperatures,
175 using either water or methanol as the permeable component, are shown in Fig. 3. Simula-
176 tions were carried out here for mole fractions above about 0.1 of the permeable component i
177 in the mixture phase. At lower mole fractions of the permeable component, the statistics for
178 the undisturbed mixture phase becomes poor for the present scenario. Fig. 3 shows that the
179 osmotic pressure rises to very high values at low concentrations of the permeable component.
180 Comparing the simulation results at the two temperatures shows that the temperature de-
181 pendence of the osmotic pressures is very small. When evaluating activity coefficients from
182 the OPAS simulations, the osmotic pressures are the upper bounds for the integration over
183 the pressure in Eq. (9).

184 In order to evaluate the OPAS simulations to get activity coefficients, the partial molar
185 volume of the respective permeable component was evaluated analytically at the composi-
186 tion of interest from the Redlich-Kister type correlations presented in Fig. 2. The resulting
187 series of values at different pressures $v_i(p)$ was then fit to a polynomial of second degree.

188 This polynomial was integrated over the pressure in order to obtain the activity coefficients
189 according to Eq. (9). The results are shown in Fig. 4 for both studied temperatures. The
190 error bars for the activity coefficient obtained with the OPAS method are within symbol
191 size. The OPAS data are smooth for both temperatures, and in the Supplementary Material,
192 it is shown that the numbers for γ_{W} and γ_{MeOH} obtained from independent OPAS simula-
193 tions are thermodynamically consistent. For $T = 323.15$ K, activity coefficients were also
194 determined from sampling the chemical potential by thermodynamic integration in NpT
195 simulations. The results are presented in Fig. 4, right panel, and in Table III. The data
196 obtained by thermodynamic integration scatter much more strongly than those from the
197 OPAS method, even though very long simulations were carried out, cf. Appendix A 3. At
198 $T = 298.15$ K, thermodynamic integration breaks down due to the increased density. In
199 contrast, the increased density at low temperatures has no effect on the performance of the
200 OPAS method. Taking into account the uncertainties in the results obtained from thermo-
201 dynamic integration, both simulation techniques agree well.

202 The data obtained from the investigated models qualitatively agree with the experimental
203 results. However, together with the original Lorentz-Berthelot rules these models are not
204 able to predict the activity coefficients quantitatively. Improvements could be achieved by
205 parameter fitting, which is beyond the scope of the present work.

206 IV. CONCLUSIONS

207 An extension of the OPAS method is presented. It is based on the simulation of an osmotic
208 equilibrium and enables accurate calculations of activity coefficients even in dense, strongly
209 interacting mixtures. For every component i in the mixture, a set of OPAS simulations
210 has to be carried out treating that component i as the permeable one. In the simulations,

211 the temperature T and the pressure p' in the pure component phase are specified and the
212 activity coefficient γ_i at the composition of the mixture phase and at T and p' is determined.
213 As high osmotic pressures are obtained, especially in simulations at high dilution of the
214 respective permeable component, the pressure dependence of the activity coefficients has to
215 be taken into account, so that the partial molar volume v_i of component i in the mixture
216 has to be known. It is obtained here from simple and inexpensive NpT simulations. The
217 proposed procedure yields activity coefficients with low uncertainty and is very robust. It is
218 an advantage of the method that its efficiency does not depend on the studied statepoint, so
219 that it is applicable to systems of low temperature and high density without modifications.
220 Furthermore, it may also be applied to solutions of polymers or proteins, for which algorithms
221 based on particle insertion become very tedious. Using OPAS simulations, the activity of the
222 solvent can be determined with high accuracy. The activity of the solute is then available
223 from the Gibbs-Duhem equation, an approach applied successfully in our previous work on
224 electrolyte solutions.

225 **SUPPLEMENTARY MATERIAL**

226 The supplementary material contains a comparison of the partial molar volumes from
227 molecular simulation and experiment as well as a consistency check of the activity coefficients
228 determined from independent OPAS simulation runs.

229 **ACKNOWLEDGMENTS**

230 The authors gratefully acknowledge financial support within the Reinhart Koselleck Pro-
231 gram (HA1993/15-1) of the German Research Foundation (DFG). The present work was

232 conducted under the auspices of the Boltzmann-Zuse Society of Computational Molecular
233 Engineering (BZS) and the simulations were carried out on the Regional University Comput-
234 ing Center Kaiserslautern (RHRK) under the grant TUKL-TLMV, the High Performance
235 Computing Center Stuttgart (HLRS) under the grant MMHBF as well as the Leibniz Su-
236 percomputing Centre (LRZ) under the grant SPARLAMPE (pr48te). The authors thank
237 Stephan Deublein and Steffen Reiser for fruitful discussions.

238 **Appendix A: Simulation Details**

239 All simulations of the present work were carried out with an extended version of the
240 molecular simulation program *ms2*⁴⁶. All simulations employed the velocity scaling ther-
241 mostat, and electrostatic long-range interactions were calculated using the reaction field
242 method^{47,48} with tinfoil boundary conditions. In all simulations, the electrostatic cutoff ra-
243 dius was equal to the Lennard-Jones cutoff radius of 15 Å. In all molecular dynamics (MD)
244 simulations, the time step was 1.2 fs, and the pressure was kept constant in MD NpT simu-
245 lations using Andersen’s barostat. Statistical simulation uncertainties were estimated with
246 the block average method by Flyvbjerg and Petersen⁴⁹.

247 **1. OPAS Simulations**

248 OPAS MD simulations were carried out as in previous work^{4,5}, using 4000 particles. At
249 first, simulations in a modified NpT ensemble were carried out, in which equilibration and
250 production took 2,000,000 and 5,000,000 time steps, respectively. The resulting box volume
251 V was then used for a run in the NVT ensemble, in which equilibration and production
252 took 2,000,000 and 10,000,000 time steps, respectively.

253 **2. Simulations for Molar Excess Volumes**

254 Conventional MD simulations were carried out in the NpT ensemble with 1000 particles.
255 Equilibration and production took 100,000 and 2,000,000 time steps, respectively.

256 **3. Simulations using Thermodynamic Integration**

The chemical potential of both components was sampled with thermodynamic integration in Monte Carlo (MC) simulations in the NpT ensemble with 1000 particles, employing an adaptive sampling technique with non-linear scaling as proposed by Kristóf and Rutkai⁴⁵ (using 100 bins from $0.2 \leq \lambda \leq 1$, with a maximum λ displacement per attempt of 0.1 and the exponent $d = 4$). These simulations yield the reduced residual chemical potential

$$\tilde{\mu}_i = [\mu_i - \mu_i^{\text{id}}(T)]/(RT), \quad (\text{A1})$$

where $\mu_i^{\text{id}}(T)$ is the part of the ideal chemical potential of component i that only depends on temperature. Activity coefficients are obtained from

$$\ln \gamma_i = \tilde{\mu}_i - \tilde{\mu}_i^{\text{pure}} - \ln x_i. \quad (\text{A2})$$

257 The fluid was equilibrated for 50,000 MC loops, before sampling was carried out for 9,600,000
258 MC loops. Each MC loop consisted of $N_{\text{NDF}}/3$ steps, where N_{NDF} indicates the total number
259 of mechanical degrees of freedom of the system, plus one attempt to resize the box according
260 to the Metropolis acceptance criterion.

261 **REFERENCES**

262 ¹B Widom. Some Topics in the Theory of Fluids. *J. Chem. Phys.*, 39:2808–2812, 1963.

- 263 ²M. P. Allen and D. J. Tildesley. *Computer Simulation of Liquids*. Clarendon Press, Oxford,
264 1991.
- 265 ³D. Frenkel and B. Smit. *Understanding Molecular Simulation: From Algorithms to Appli-*
266 *cations, 2nd ed.* Academic Press, Elsevier, San Diego, 2002.
- 267 ⁴M. Kohns, S. Reiser, M. Horsch, and H. Hasse. Solvent activity in electrolyte solutions
268 from molecular simulation of the osmotic pressure. *J. Chem. Phys.*, 144:084112, 2016.
- 269 ⁵M. Kohns, M. Schappals, M. Horsch, and H. Hasse. Activities in Aqueous Solutions of the
270 Alkali Halide Salts from Molecular Simulation. *J. Chem. Eng. Data*, 61:4068–4076, 2016.
- 271 ⁶J. Powles, S. Murad, and P. Ravi. A New Model for Permeable Micropores. *Chem. Phys.*
272 *Lett.*, 188:21–24, 1992.
- 273 ⁷S. Murad and J. Powles. A Computer Simulation of the Classic Experiment on Osmosis
274 and Osmotic Pressure. *J. Chem. Phys.*, 99:7271–7272, 1993.
- 275 ⁸S. Murad, J. Powles, and B. Holtz. Osmosis and reverse osmosis in solutions: Monte Carlo
276 simulations and van der Waals one-fluid theory. *Mol. Phys.*, 86:1473–1483, 1995.
- 277 ⁹F. Paritosh and S. Murad. Molecular simulations of osmosis and reverse osmosis in aqueous
278 electrolyte solutions. *AIChE J.*, 42:2984–2986, 1996.
- 279 ¹⁰P. S. Crozier and R. L. Rowley. Activity coefficient prediction by osmotic molecular
280 dynamics. *Fluid Phase Equilib.*, 193:53–73, 2002.
- 281 ¹¹*Dortmund Data Bank, Version 2016*. DDBST GmbH, Oldenburg, 2016.
- 282 ¹²S. K. Allison, J. P. Fox, R. Hargreaves, and S. P. Bates. Clustering and microimmiscibility
283 in alcohol-water mixtures: Evidence from molecular-dynamics simulations. *Phys. Rev. B*,
284 71:024201, 2005.
- 285 ¹³I. Bako, T. Megyes, S. Balint, T. Grosz, and V. Chihaiia. Water-methanol mixtures:
286 topology of hydrogen bonded network. *Phys. Chem. Chem. Phys.*, 10:5004–5011, 2008.

- 287 ¹⁴A. A. Chialvo and P. T. Cummings. Structure of Mixed-Solvent Electrolyte Solutions via
288 Gibbs Ensemble Monte Carlo Simulation. *Mol. Sim.*, 11:163–175, 1993.
- 289 ¹⁵L. Dougan, R. Hargreaves, S. P. Bates, J. L. Finney, V. Reat, A. K. Soper, and J. Crain.
290 Segregation in aqueous methanol enhanced by cooling and compression. *J. Chem. Phys.*,
291 122:174514, 2005.
- 292 ¹⁶M. Ferrario, M. Haughney, I. R. McDonald, and M. L. Klein. Molecular Dynamics Simula-
293 tion of Aqueous Mixtures - Methanol, Acetone, and Ammonia. *J. Chem. Phys.*, 93:5156–
294 5166, 1990.
- 295 ¹⁷L. C. G. Freitas. Monte Carlo Simulation of the Binary Liquid Mixture Water-Methanol.
296 *J. Mol. Struct. (Theochem)*, 101:151–158, 1993.
- 297 ¹⁸E. Galicia-Andrés, H. Dominguez, L. Pusztai, and O. Pizio. Composition dependence
298 of thermodynamic, dynamic and dielectric properties of water-methanol model mixtures.
299 Molecular dynamics simulation results with the OPLS-AA model for methanol. *J. Mol.*
300 *Liq.*, 212:70–78, 2015.
- 301 ¹⁹D. Gonzalez-Salgado and I. Nezbeda. Excess properties of aqueous mixtures of methanol:
302 Simulation versus experiment. *Fluid Phase Equilib.*, 240:161–166, 2006.
- 303 ²⁰D. Gonzalez-Salgado, K. Zemankova, E. G. Noya, and E. Lomba. Temperature of max-
304 imum density and excess thermodynamics of aqueous mixtures of methanol. *J. Chem.*
305 *Phys.*, 144:184505, 2016.
- 306 ²¹G. Guevara-Carrion, J. Vrabec, and H. Hasse. Prediction of self-diffusion coefficient and
307 shear viscosity of water and its binary mixtures with methanol and ethanol by molecular
308 simulation. *J. Chem. Phys.*, 134:074508, 2011.
- 309 ²²G. Guevara-Carrion, Y. Gaponenko, T. Janzen, J. Vrabec, and V. Shevtsova. Diffusion
310 in Multicomponent Liquids: From Microscopic to Macroscopic Scales. *J. Phys. Chem. B.*,

311 120:12193–12210, 2016.

312 ²³S Hempel, J Fischer, and G Paschek, D Sadowski. Activity Coefficients of Complex
313 Molecules by Molecular Simulation and Gibbs-Duhem Integration. *Soft Mater.*, 10:26–41,
314 2012.

315 ²⁴W. L. Jorgensen and J. D. Madura. Quantum and Statistical Studies of Liquids 25:
316 Solvation and Conformation of Methanol in Water. *J. Am. Chem. Soc.*, 105:1407–1413,
317 1983.

318 ²⁵A. Laaksonen, P. G. Kusalik, and I. M. Svishchev. Three-dimensional structure in water-
319 methanol mixtures. *J. Phys. Chem. A*, 101:5910–5918, 1997.

320 ²⁶M. Lisal, W. R. S. William, and I. Nezbeda. Accurate vapour-liquid equilibrium calcu-
321 lations for complex systems using the reaction Gibbs ensemble Monte Carlo simulation
322 method. *Fluid Phase Equilib.*, 181:127–146, 2001.

323 ²⁷F. Moučka and I. Nezbeda. Water-methanol mixtures with non-Lorentz-Berthelot com-
324 bining rules: A feasibility study. *J. Mol. Liq.*, 159:47–51, 2011.

325 ²⁸C. Nieto-Draghi, R. Hargreaves, and S. P. Bates. Structure and dynamics of water in
326 aqueous methanol. *J. Phys.: Condens. Matter*, 17:S3265–S3272, 2005.

327 ²⁹S. Y. Noskov, M. G. Kiselev, A. M. Kolker, and B. M. Rode. Structure of methanol-
328 methanol associates in dilute methanol-water mixtures from molecular dynamics simula-
329 tion. *J. Mol. Liq.*, 91:157–165, 2001.

330 ³⁰S. Okazaki, K. Nakanishi, and H. Touhara. Computer Experiments on Aqueous Solutions
331 1: Monte Carlo Calculation on the Hydration of Methanol in an Infinitely Dilute Aqueous
332 Solution with a New Water-Methanol Pair Potential. *J. Chem. Phys.*, 78:454–469, 1983.

333 ³¹S. Okazaki, H. Touhara, and K. Nakanishi. Computer Experiments on Aqueous Solutions
334 5: Monte Carlo Calculation on the Hydrophobin Interaction in 5 Mol-Percent Methanol

335 Solution. *J. Chem. Phys.*, 81:890–894, 1984.

336 ³²T. Ono, K. Horikawa, M. Ota, Y. Sato, and H. Inomata. Insight into the Local Com-
337 position of the Wilson Equation at High Temperatures and Pressures through Molecular
338 Simulations of Methanol-Water Mixtures. *J. Chem. Eng. Data*, 59:1024–1030, 2014.

339 ³³T. A. Pascal and W. A. Goddard, III. Hydrophobic Segregation, Phase Transitions and the
340 Anomalous Thermodynamics of Water/Methanol Mixtures. *J. Phys. Chem. B*, 116:13905–
341 13912, 2012.

342 ³⁴J.-C. Soetens and P. A. Bopp. Water-Methanol Mixtures: Simulations of Mixing Properties
343 over the Entire Range of Mole Fractions. *J. Phys. Chem. B*, 119:8593–8599, 2015.

344 ³⁵H. J. Strauch and P. T. Cummings. Gibbs Ensemble Simulation of Mixed-Solvent Elec-
345 trolyte Solutions. *Fluid Phase Equilib.*, 86:147–172, 1993.

346 ³⁶H. Tanaka and K. E. Gubbins. Structure and Thermodynamic Properties of Water-
347 Methanol Mixtures - Role of the Water-Water Interaction. *J. Chem. Phys.*, 97:2626–2634,
348 1992.

349 ³⁷H. Tanaka, J. Walsh, and K. E. Gubbins. Structure of Water-Methanol Binary Mixtures
350 - Role of the Water-Water Interaction. *Mol. Phys.*, 76:1221–1233, 1992.

351 ³⁸L. Vlcek and I. Nezbeda. Excess properties of aqueous mixtures of methanol: Simple
352 models versus experiment. *J. Mol. Liq.*, 131:158–162, 2007.

353 ³⁹E. J. W. Wensink, A. C. Hoffmann, P. J. van Maaren, and D. van der Spoel. Dynamic
354 properties of water/alcohol mixtures studied by computer simulation. *J. Chem. Phys.*,
355 119:7308–7317, 2003.

356 ⁴⁰H. Yu, D. P. Geerke, H. Liu, and W. E. van Gunsteren. Molecular dynamics simulations of
357 liquid methanol and methanol-water mixtures with polarizable models. *J. Comput. Chem.*,
358 27:1494–1504, 2006.

359 ⁴¹Y. Zhong, G. L. Warren, and S. Patel. Thermodynamic and structural properties
360 of methanol-water solutions using nonadditive interaction models. *J. Comput. Chem.*,
361 29:1142–1152, 2008.

362 ⁴²J. L. F. Abascal and C. Vega. A general purpose model for the condensed phases of water:
363 TIP4P/2005. *J. Chem. Phys.*, 123:234505, 2005.

364 ⁴³T. Schnabel, A. Srivastava, J. Vrabec, and H. Hasse. Hydrogen bonding of methanol
365 in supercritical CO₂: Comparison between H-1 NMR spectroscopic data and molecular
366 simulation results. *J. Phys. Chem. B*, 111:9871–9878, 2007.

367 ⁴⁴W. R. Smith, F. Moucka, and I. Nezbeda. Osmotic pressure of aqueous electrolyte solu-
368 tions via molecular simulations of chemical potentials: Application to NaCl. *Fluid Phase*
369 *Equilib.*, 407:76–83, 2016.

370 ⁴⁵T. Kristóf and G. Rutkai. Chemical potential calculations by thermodynamic integration
371 with separation shifting in adaptive sampling Monte Carlo simulations. *Chem. Phys. Lett.*,
372 445:74–78, 2007.

373 ⁴⁶C. W. Glass, S. Reiser, G. Rutkai, S. Deublein, A. Köster, G. Guevara-Carrión, A. Wafai,
374 M. Horsch, M. Bernreuther, T. Windmann, H. Hasse, and J. Vrabec. ms2: A Molecular
375 Simulation Tool for Thermodynamic Properties, New Version Release. *Comput. Phys.*
376 *Commun.*, 185:3302–3306, 2014.

377 ⁴⁷J. A. Barker and R. O. Watts. Monte Carlo Studies of Dielectric Properties of Water-like
378 Models. *Mol. Phys.*, 26:789–792, 1973.

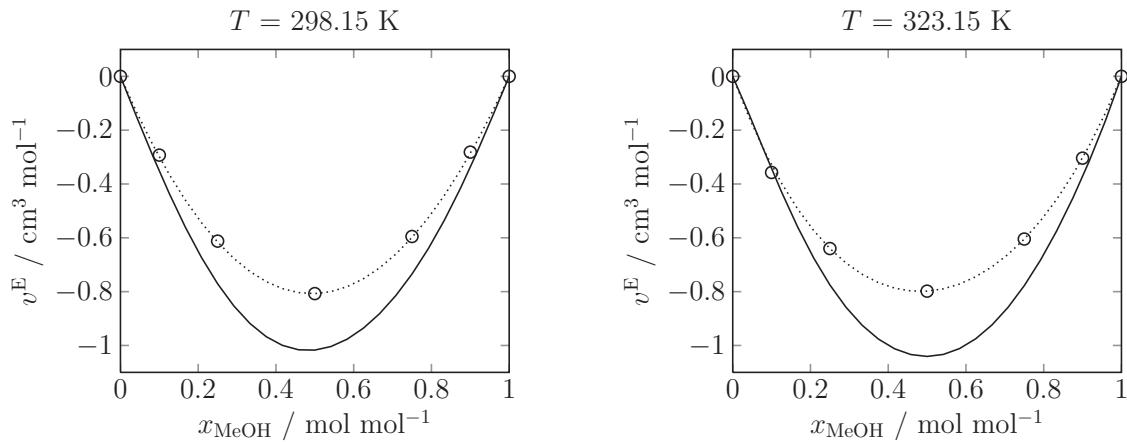
379 ⁴⁸B. Saager, J. Fischer, and M. Neumann. Reaction Field Simulations of Monatomic and
380 Diatomic Dipolar Fluids. *Mol. Sim.*, 6:27–49, 1991.

381 ⁴⁹H. Flyvbjerg and H.G. Petersen. Error-estimates on Averages of Correlated Data. *J.*
382 *Chem. Phys.*, 91:461–466, 1989.

- 383 ⁵⁰C. Coquelet, A. Valtz, and D. Richon. Volumetric properties of water plus mo-
384 noethanolamine plus methanol mixtures at atmospheric pressure from 283.15 to 353.15
385 K. *J. Chem. Eng. Data*, 50:412–418, 2005.
- 386 ⁵¹W. Wagner and A. Pruss. The IAPWS formulation 1995 for the thermodynamic properties
387 of ordinary water substance for general and scientific use. *J. Phys. Chem. Ref. Data*,
388 31:387–535, 2002.
- 389 ⁵²K. de Reuck and R. Craven. *Methanol, International Thermodynamic Tables of the Fluid*
390 *State - 12*. IUPAC, Blackwell Scientific Publications, London, 1993.
- 391 ⁵³Z. S. Kooner, R. C. Phutela, and D. V. Fenby. Determination of the Equilibrium Constants
392 of Water-Methanol Deuterium Exchange Reactions From Vapor Pressure Measurements.
393 *Aust. J. Chem.*, 33:9–13, 1980.
- 394 ⁵⁴S. Bernatova, K. Aim, and I. Wichterle. Isothermal vapour-liquid equilibrium with chem-
395 ical reaction in the quaternary water plus methanol plus acetic acid plus methyl acetate
396 system, and in five binary sub-systems. *Fluid Phase Equilib.*, 247:96–101, 2006.

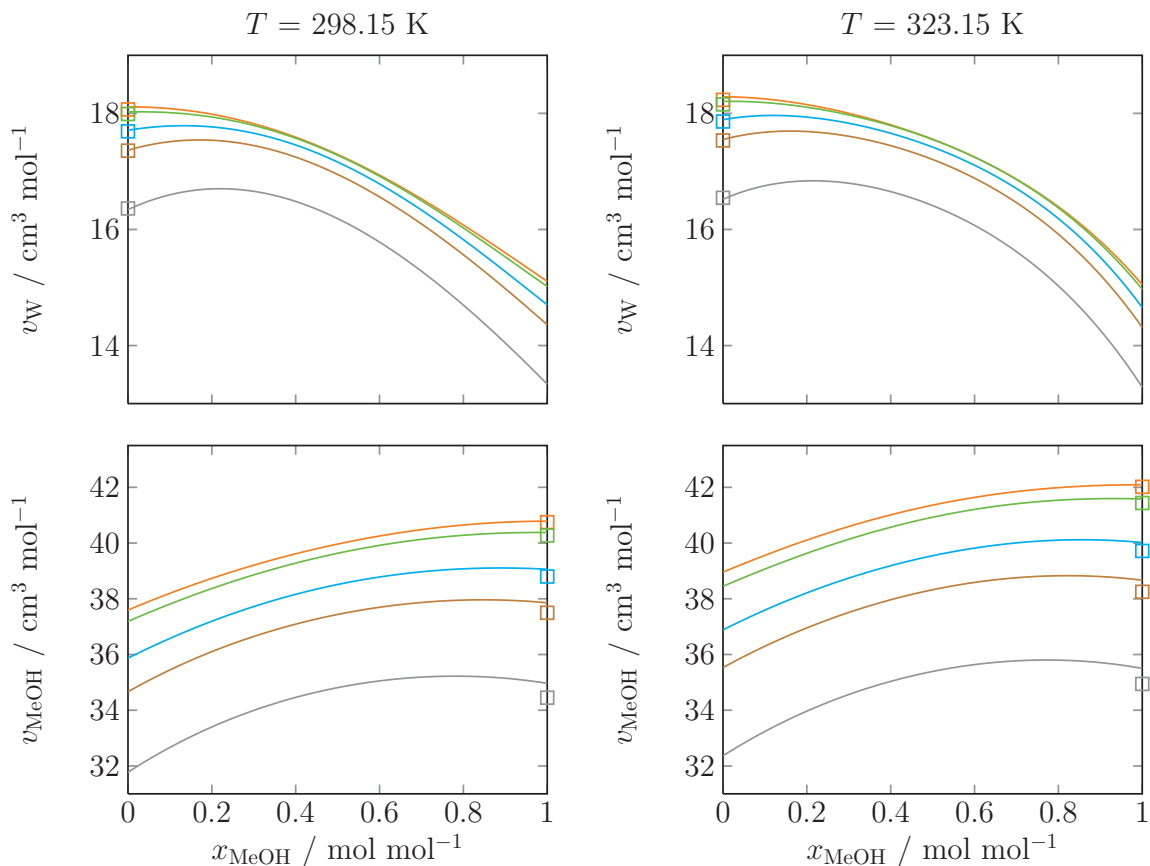
397 **FIGURES**

398



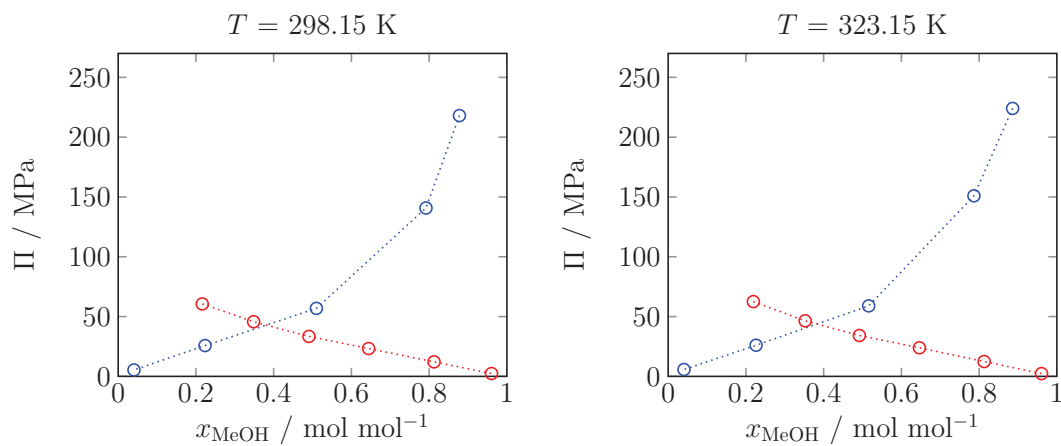
399

400 FIG. 1. Molar excess volume of water-methanol mixtures at $p = 1$ bar. Open circles show re-
 401 sults from predictive molecular simulations, dotted lines show a Redlich-Kister correlation to these
 402 results. Statistical uncertainties are within symbol size. The solid lines are correlations to experi-
 403 mental data by Coquelet et al.⁵⁰.



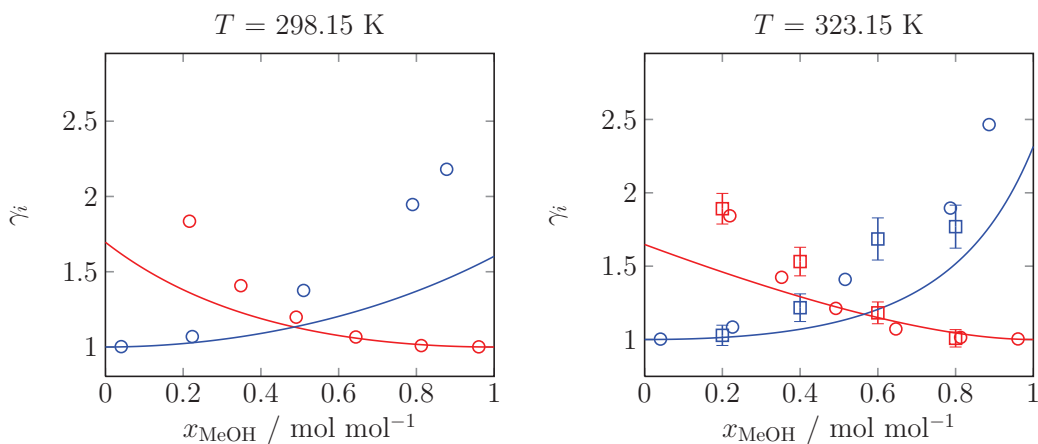
404

405 FIG. 2. Partial molar volumes of water (top) and methanol (bottom) in water-methanol mixtures
 406 obtained from Redlich-Kister correlations fit to molecular simulation results for the molar excess
 407 volumes. Colored lines correspond to different pressures (from top to bottom): (-) $p = 0.1$ MPa,
 408 (-) $p = 10$ MPa, (-) $p = 50$ MPa, (-) $p = 100$ MPa, (-) $p = 300$ MPa. The open squares colored
 409 accordingly are molar volumes of pure water and pure methanol computed from the equations of
 410 state by Wagner and Pruss⁵¹ and by de Reuck and Craven⁵², respectively.



411

412 FIG. 3. Osmotic pressures obtained in OPAS simulations using water as the permeable component
 413 (blue) or methanol as the permeable component (red), dotted lines are guides to the eye. Statistical
 414 uncertainties are within symbol size.



415

416 FIG. 4. Activity coefficients of methanol (red) and water (blue) in water-methanol mixtures at
 417 $p = 1 \text{ bar}$. The open circles show simulation results obtained with the OPAS method. Statistical
 418 uncertainties of the OPAS simulation results are within symbol size. In the right panel, the open
 419 squares show simulation results using the same molecular models, but obtained from sampling the
 420 chemical potential with thermodynamic integration. Solid lines show NRTL fits to the experimental
 421 data of Kooner et al.⁵³ (left panel) and by Bernatova et al.⁵⁴ (right panel).

422 **TABLES**

423

424 TABLE I. Densities and parameters of Redlich-Kister correlations, cf. Eq. (12), fit to molar excess
 425 volumes for binary mixtures of water and methanol.

		$\rho(T = 298.15 \text{ K}) / \text{mol l}^{-1}$					$T = 298.15 \text{ K}$		
		$x_{\text{MeOH}} / \text{mol mol}^{-1}$					Parameters for $v^{\text{E}} / \text{cm}^3 \text{mol}^{-1}$		
p / MPa	0	0.25	0.5	0.75	1	A_0	A_1	A_2	
0.1	55.21(1)	43.16(1)	34.92(1)	28.968(6)	24.519(4)	-3.2292	-0.0916	0.0372	
10	55.48(1)	43.384(9)	35.126(8)	29.203(6)	24.766(5)	-2.9315	-0.0829	-0.1153	
50	56.47(1)	44.149(9)	35.907(6)	30.021(5)	25.602(4)	-2.1386	0.0819	-0.0695	
100	57.59(1)	45.012(8)	36.757(6)	30.850(5)	26.417(3)	-1.6183	0.2426	0.1887	
300	61.194(8)	47.756(6)	39.197(5)	33.136(4)	28.600(2)	-0.5657	0.3927	0.2493	
		$\rho(T = 323.15 \text{ K}) / \text{mol l}^{-1}$					$T = 323.15 \text{ K}$		
		$x_{\text{MeOH}} / \text{mol mol}^{-1}$					Parameters for $v^{\text{E}} / \text{cm}^3 \text{mol}^{-1}$		
p / MPa	0	0.25	0.5	0.75	1	A_0	A_1	A_2	
0.1	54.69(1)	42.259(9)	33.973(7)	28.123(6)	23.758(5)	-3.0137	0.0507	-0.2649	
10	54.93(1)	42.517(8)	34.228(8)	28.390(5)	24.048(4)	-2.7153	-0.0796	-0.2904	
50	55.896(9)	43.378(7)	35.138(6)	29.310(4)	24.989(3)	-1.9799	-0.0070	0.0629	
100	56.99(1)	44.314(6)	36.047(5)	30.205(4)	25.862(3)	-1.4650	0.1003	0.0757	
300	60.538(6)	47.157(5)	38.641(4)	32.639(3)	28.169(2)	-0.5199	0.3133	0.2183	

426 TABLE II. Results of OPAS simulations and deduced activity coefficients. The upper block shows
 427 results of simulations with methanol as the permeable component, the lower block shows results of
 428 simulations with water as the permeable component.

$T = 298.15 \text{ K}$			$T = 323.15 \text{ K}$		
$x_{\text{MeOH}} / \text{mol mol}^{-1}$	Π / MPa	γ_{MeOH}	$x_{\text{MeOH}} / \text{mol mol}^{-1}$	Π / MPa	γ_{MeOH}
0.2163(5)	60.6(3)	1.84(1)	0.2192(6)	62.5(3)	1.84(1)
0.3484(5)	45.7(3)	1.407(6)	0.3525(5)	46.4(2)	1.42(6)
0.4907(5)	33.4(2)	1.198(4)	0.4919(4)	34.2(2)	1.212(4)
0.6442(2)	23.3(2)	1.067(3)	0.6463(2)	23.9(1)	1.072(2)
0.8126(1)	12.1(1)	1.009(2)	0.8129(1)	12.4(1)	1.014(2)
0.9605(1)	2.382(5)	1.0011(8)	0.9606(1)	2.33(5)	1.0039(8)
$x_{\text{MeOH}} / \text{mol mol}^{-1}$	Π / MPa	γ_{W}	$x_{\text{MeOH}} / \text{mol mol}^{-1}$	Π / MPa	γ_{W}
0.8779(8)	218.0(5)	2.18(1)	0.8862(8)	224.1(5)	2.46(1)
0.7901(8)	140.9(4)	1.946(7)	0.7866(8)	151.0(5)	1.896(7)
0.5098(7)	56.9(4)	1.376(4)	0.5159(7)	58.9(3)	1.410(3)
0.2235(4)	25.8(2)	1.069(2)	0.2262(4)	26.0(2)	1.085(2)
0.0407(1)	5.36(9)	1.0025(7)	0.0408(1)	5.72(9)	1.0028(7)

429 TABLE III. Reduced residual chemical potentials and activity coefficients of water and methanol
 430 at $T = 323.15$ K and $p = 1$ bar obtained from thermodynamic integration.

$x_{\text{MeOH}} / \text{mol mol}^{-1}$	$\tilde{\mu}_{\text{MeOH}}$	$\tilde{\mu}_{\text{W}}$	γ_{MeOH}	γ_{W}
0	—	-10.40(6)	—	1
0.2	-8.68(6)	-10.60(7)	1.9(1)	1.03(7)
0.4	-8.20(6)	-10.72(8)	1.5(1)	1.22(9)
0.6	-8.05(6)	-10.79(9)	1.18(7)	1.7(1)
0.8	-7.92(6)	-11.44(8)	1.01(6)	1.8(1)
1	-7.71(6)	—	1	—

Supplementary Material to:

Activity Coefficients from Molecular Simulations using the OPAS Method

Maximilian Kohns, Martin Horsch, and Hans Hasse

1 Comparison of Partial Molar Volume Results to Experimental Data

In the literature, two sets of experimental data on densities or molar excess volumes of mixtures of water and methanol at elevated pressures exist: Safarov et al. [1] studied the system at pressures up to $p = 60$ MPa and report data for $T = 298.15$ K and $T = 323.15$ K. Yokoyama et al. [2] studied the system at pressures up to $p = 200$ MPa, but report data only at $T = 320$ K and higher temperatures. However, $T = 320$ K seems close enough to $T = 323.15$ K to enable at least a qualitative comparison with our molecular simulation predictions.

Both groups only give numbers for the densities or molar volumes, respectively. Thus, they were correlated with a Redlich-Kister approach, similar to the treatment of the molecular simulation data discussed in the main text, and the partial molar volumes were computed from these correlations. To enable this, the data of Safarov et al. [1] were complemented with the pure component molar volumes computed from the equations of state by Wagner and Pruss [3] for water and de Reuck and Craven [4] for methanol, respectively. Yokoyama et al. [2] report data for pure methanol, which were used here, and their data set was also complemented

with the equation of state of Wagner and Pruss [3] for water. The partial molar volumes obtained in this manner are shown in Figs. S1 and S2, respectively, where they are compared to the predictions from molecular simulations already discussed in the main text.

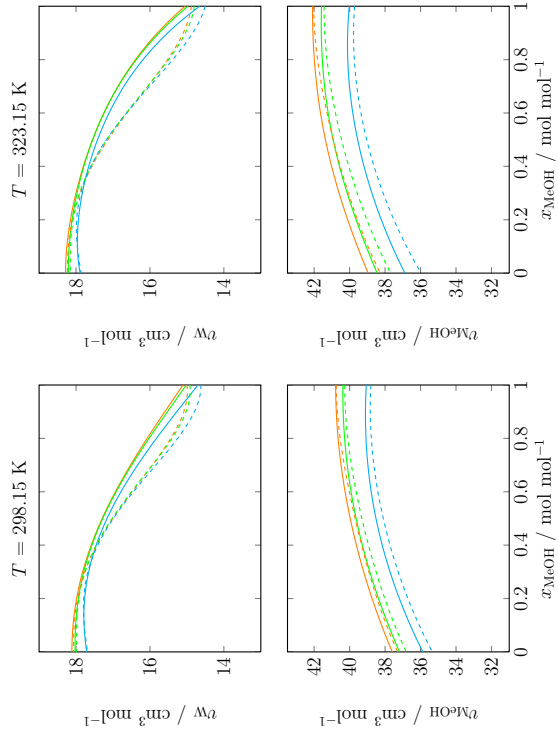


Figure S1: Partial molar volumes of water (top) and methanol (bottom) in water-methanol mixtures obtained from Redlich-Kister correlations. Solid lines: fit to molecular simulation results for the molar excess volumes. Dashed lines: Fit to experimental data by Safarov et al. [1]. Colors correspond to different pressures (from top to bottom): (—) $p = 0.1$ MPa, (—) $p = 10$ MPa, (—) $p = 50$ MPa.

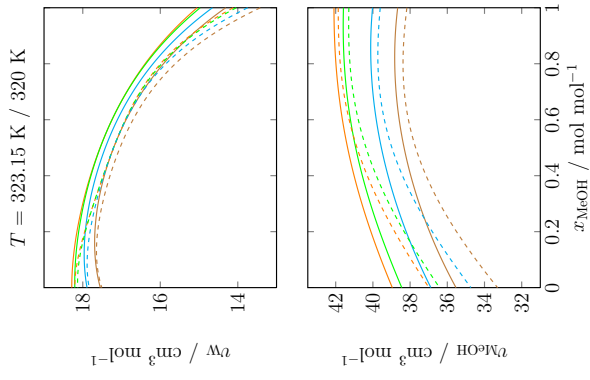


Figure S2: Partial molar volumes of water (top) and methanol (bottom) in water-methanol mixtures obtained from Redlich-Kister correlations. Solid lines: fit to molecular simulation results for the molar excess volumes, $T = 323.15$ K. Dashed lines: Fit to experimental data by Yokoyama et al. [2], $T = 320$ K. Colors correspond to different pressures (from top to bottom): (-) $p = 0.1$ MPa, (-) $p = 10$ MPa, (-) $p = 50$ MPa, (-) $p = 100$ MPa.

The molecular simulation predictions of v_{MeOH} (bottom panels in both figures) compare favorably with both sets of experimental data. The qualitative dependence of v_{MeOH} on x_{MeOH} is predicted correctly. This is true for all isobars. However, the molecular simulation predictions show a quantitative offset from the experimental data, which mainly arises from the differences in $v_{\text{MeOH}}^{\text{pure}}$ already discussed in the main text. Additionally, the partial molar volume of methanol at infinite dilution in water v_{MeOH}^{∞} seems to be slightly underpredicted.

Regarding v_{W} (top panels in both figures), the agreement between the molecular simulation predictions and the experimental data is very good in the water-rich region. In the methanol-rich region, however, the two sets of experimental data disagree. It is therefore difficult to assess the molecular simulation predictions. If compared to the experimental data of Safarov et al. [1] (Fig. S1), the numbers for the partial molar volume of water at infinite dilution in methanol v_{W}^{∞} are predicted correctly, but the dependence of v_{W} on x_{MeOH} is not depicted correctly. If compared to the experimental data of Yokoyama et al. [2] (Fig. S2), the prediction of the trend is very good, whereas the numbers found for v_{MeOH}^{∞} are too low.

Altogether, taking into account that the employed molecular models are conceptually simple and no adjustment to experimental data of the binary system was made, the predictions of the partial molar volumes are quite satisfactory.

2 NRTL Fit to Simulation Results

In order to check the thermodynamic consistency of the values for γ_W and γ_{MeOH} obtained from independent OPAS simulation runs, an NRTL model was fit to the data, cf. Fig. S3. For both temperatures, the data and the fits agree well, which shows the consistency of the OPAS simulation results.

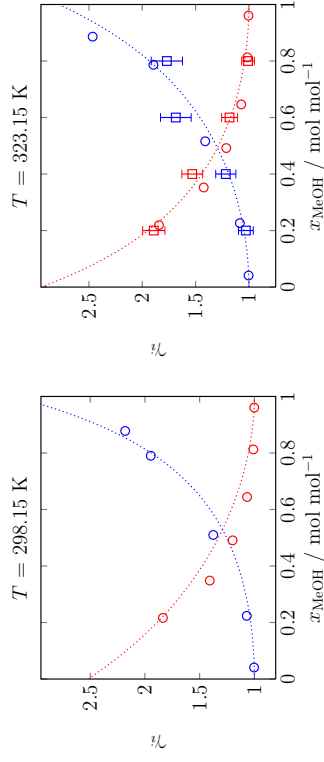


Figure S3: Activity coefficients of methanol (red) and water (blue) in water-methanol mixtures at $p = 1$ bar. The open circles show simulation results obtained with the OPAS method. Statistical uncertainties of the OPAS simulation results are within symbol size. In the right panel, the open squares show simulation results using the same molecular models, but obtained from sampling the chemical potential with thermodynamic integration. Dotted lines show a simultaneous fit to the OPAS simulation results for the activity coefficients of both components using an NRTL model.

References

- [1] J. Safarov, S. Heydarov, A. Shahverdiyev, and E. Hassel. Excess molar volumes V_m^E , isothermal compressibilities k , and thermal expansivities α of $\{(1-x)\text{H}_2\text{O}+x\text{CH}_3\text{OH}\}$ at $T = (298.15 \text{ to } 523.15) \text{ K}$ and pressures up to 60 MPa. *J. Chem. Thermodynamics*, 36:541–547, 2004.
- [2] H. Yokoyama and M. Uematsu. Thermodynamic properties of $\{x\text{CH}_3\text{OH}+(1-x)\text{H}_2\text{O}\}$ at $x = (1.0000, 0.8005, 0.4002, \text{ and } 0.2034)$ in the temperature range from 320 K to 420 K at pressures up to 200 MPa. *J. Chem. Thermodynamics*, 35:813–823, 2003.
- [3] W. Wagner and A. Pruss. The IAPWS formulation 1995 for the thermodynamic properties of ordinary water substance for general and scientific use. *J. Phys. Chem. Ref. Data*, 31:387–535, 2002.
- [4] K. de Reuck and R. Craven. *Methanol, International Thermodynamic Tables of the Fluid State - 12*. IUPAC, Blackwell Scientific Publications, London, 1993.