

Molecular Simulation Study of the CO₂-N₂O Analogy

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Abstract

The validity of the CO₂-N₂O analogy concerning the Henry's law constant is investigated by molecular simulation of the solvents water, ethanol, as well as their mixtures. Molecular models for carbon dioxide (CO₂) and for the solvents water and ethanol are taken from the literature. For nitrous oxide (N₂O), two new molecular models are presented. They differ in their structure, but are both parametrized using pure component vapor-liquid equilibrium data. The models are used to study Henry's law constants of CO₂ and N₂O in pure water and pure ethanol over a wide range of temperatures. In the case of water, a slight adjustment in the water-solute interaction is necessary to achieve agreement with experimental data, whereas the gas solubilities in ethanol are predicted quantitatively without any adjustment. For

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mixed solvents containing both water and ethanol, the CO₂-N₂O analogy is found to be invalid, contradicting a widespread assumption.

Keywords: CO₂, N₂O, Molecular model, Vapor-liquid equilibrium, Henry's law constant

1. Introduction

The removal of carbon dioxide (CO₂) from gas streams is a common task in many industrial processes. Large scale applications are for example post combustion carbon capture processes and natural gas cleaning. The common process for CO₂ removal is reactive absorption with aqueous solutions of amines as solvents [1, 2]. Upon adsorption, CO₂ reacts with these solvents, forming carbonates and eventually carbamates. CO₂-loaded aqueous amine solutions may show complex phase behavior [3].

In the design of reactive absorption processes, the physical solubility of CO₂ in the solvent is a key property, as it is needed to describe both the phase equilibrium and the chemical equilibrium. A measure of the physical gas solubility is the Henry's law constant $H_{i,j}$ of the solute i in the solvent j . Unfortunately, for reactive systems, this property cannot be determined experimentally in many cases, namely when chemical and physical effects cannot be disentangled in the experiment. Therefore, for reactive systems with

16 CO₂, the CO₂-N₂O analogy has been established as a workaround [4]. In
 17 contrast to CO₂, nitrous oxide (N₂O) normally does not react with the sol-
 18 vents used in reactive absorption processes, so that the Henry’s law constant
 19 of N₂O in any of these solvents can be determined experimentally using stan-
 20 dard methods. Furthermore, the Henry’s law constant of both CO₂ and N₂O
 21 in pure water is known [5, 6]. In evaluating the Henry’s law constant of CO₂
 22 in water, the fact that CO₂ is a weak electrolyte is usually neglected, since
 23 the amount of reaction products of CO₂ is very small when the solvent is pure
 24 water [7–9]. The CO₂-N₂O analogy assumes that, for a given temperature,
 25 the ratio R_H of both gases’ Henry’s law constants does not depend on the
 26 solvent composition, i.e. for aqueous amine solutions:

$$R_H = \frac{H_{\text{N}_2\text{O},\text{water}}}{H_{\text{CO}_2,\text{water}}} \stackrel{!}{=} \frac{H_{\text{N}_2\text{O},\text{aqueous amine solution}}}{H_{\text{CO}_2,\text{aqueous amine solution}}} \stackrel{!}{=} \text{const.}, \quad (1)$$

27 Relying on the validity of Eq. (1), the unknown Henry’s law constant of CO₂
 28 in an aqueous amine solution can be determined from the experimental data
 29 on the Henry’s law constant of N₂O in that solution and the Henry’s law
 30 constants of both gases in pure water.

31 The CO₂-N₂O analogy is generally considered to be a practically useful en-
 32 gineering rule [4, 10, 11]. The idea is loosely based on the similarity of the

33 two molecules CO_2 and N_2O : Both are composed of three atoms, have a lin-
34 ear structure and the same molecular mass [12]. Furthermore, many pure
35 component properties of CO_2 and N_2O , such as the vapor pressure, the satu-
36 rated liquid and vapor density, and the enthalpy of vaporization, are similar,
37 but not completely identical [13, 14]. However, these similarities in the pure
38 component properties do not imply that their behavior in mixtures is similar
39 as well. Nevertheless, the CO_2 - N_2O analogy assumes that a non-reacting
40 CO_2 molecule is similar to a N_2O molecule when gas solubilities in various
41 solvents are considered.

42 As an alternative to experimental studies, molecular simulation can be used
43 to study phase equilibria. When carried out properly, the quality of the
44 simulation results solely depends on the quality of the employed molecular
45 models. In this work, a simple approach for assessing the validity of the
46 CO_2 - N_2O analogy based on molecular simulation is presented. It is explored
47 for the solvents water, ethanol, and their mixtures. Using molecular simula-
48 tion, the Henry's law constants of CO_2 and N_2O are predicted for the pure
49 solvents and their mixtures and the validity of the CO_2 - N_2O analogy is as-
50 sessed. The solvent ethanol is chosen here because experimental data on the
51 solubility of both gases in ethanol are available in the literature [15–17]. The

52 reactions of CO_2 with ethanol are negligible, as they are for water, and also
53 N_2O does not react with ethanol. Additionally, experimental data also exist
54 in the literature for the solubility of CO_2 in mixtures of water and ethanol
55 [17]. In molecular simulations, only physical effects are considered. Hence,
56 even when a mixture is modeled in which CO_2 would react, e.g. an aqueous
57 amine solution, the reactions are neglected and only the physical solubility
58 is studied in the simulations.

59 A number of molecular simulation studies have been carried out for deter-
60 mining the solubilities of CO_2 in various solvents [18–25], whereas N_2O has
61 drawn much less attention [25]. There is a large number of CO_2 models
62 available in the literature, some of which describe the VLE of pure CO_2 rea-
63 sonably well. For a detailed discussion, see the recent paper of Jiang et al.
64 [26]. In contrast, there are only three N_2O models available to date [27–29],
65 and none of them is able to describe the VLE with satisfactory accuracy.
66 Therefore, two new molecular models for N_2O are developed in the present
67 work, which differ in their structure, but are parametrized using the same
68 data.

69 **2. Molecular Models and Simulation Methods**

70 *2.1. Molecular Models*

71 All molecular models used in the present study are rigid and non-pola-
 72 rizable. They contain Lennard-Jones (LJ) sites to describe dispersion and
 73 repulsion and partial charges or point quadrupoles to account for hydrogen
 74 bonding and polarity. Therefore, all potentials can be written as special cases
 75 of

$$\begin{aligned}
 U &= U_{\text{LJ}} + U_{\text{CC}} + U_{\text{CQ}} + U_{\text{QQ}} \\
 &= \sum_{i=1}^{N-1} \sum_{j=i+1}^N \left\{ \sum_{a=1}^{n_i^{\text{LJ}}} \sum_{b=1}^{n_j^{\text{LJ}}} 4\epsilon_{ijab} \left[\left(\frac{\sigma_{ijab}}{r_{ijab}} \right)^{12} - \left(\frac{\sigma_{ijab}}{r_{ijab}} \right)^6 \right] \right. \\
 &\quad + \sum_{c=1}^{n_i^e} \sum_{d=1}^{n_j^e} \frac{1}{4\pi\epsilon_0} \left[\frac{q_{ic}q_{jd}}{r_{ijcd}} + \frac{q_{ic}Q_{jd} + q_{jd}Q_{ic}}{r_{ijcd}^3} f_1(\omega_i, \omega_j) \right. \\
 &\quad \left. \left. + \frac{Q_{ic}Q_{jd}}{r_{ijcd}^5} \cdot f_2(\omega_i, \omega_j) \right] \right\}. \tag{2}
 \end{aligned}$$

76 Here, a , b , c and d denote sites, i and j denote molecules. ϵ_0 is the vac-
 77 uum permittivity, ϵ_{ijab} and σ_{ijab} are the Lennard-Jones energy and size pa-
 78 rameters, r_{ijab} and r_{ijcd} are site-site distances, q_{ic} , q_{jd} , Q_{ic} and Q_{jd} are the
 79 magnitude of the point charges and quadrupole moments, respectively. More-
 80 over, $f_1(\omega_i, \omega_j)$ and $f_2(\omega_i, \omega_j)$ are dimensionless angle-dependent expressions

81 in terms of the orientation (ω_i, ω_j) of the electrostatic interaction sites [30].
82 The modified Lorentz-Berthelot combining rules are used [31, 32] for the
83 interaction between unlike Lennard-Jones sites

$$\sigma_{ijab} = \frac{\sigma_{iiaa} + \sigma_{jjbb}}{2}, \quad (3)$$

$$\epsilon_{ijab} = \xi_{ij} \sqrt{\epsilon_{iiaa} \epsilon_{jjbb}}. \quad (4)$$

84 The adjustment of a binary interaction parameter ξ_{ij} is only necessary for
85 the systems 'water + CO₂' and 'water + N₂O' (cf. Section 3.2). Hence, the
86 original Lorentz-Berthelot combining rules are retained in all other cases, i.e.
87 $\xi_{ij} = 1$ for 'water + ethanol' as well as 'ethanol + CO₂' and 'ethanol + N₂O'.
88 The molecular models for the two solvents water and ethanol are taken from
89 the literature: the water model is TIP4P/2005 [33], and the ethanol model
90 is taken previous work of our group [20]. Both models have been used suc-
91 cessfully in several molecular simulation studies [34–38].
92 For CO₂, two different models are assessed, which are taken from previous
93 work of our group. The model of Vrabc et al. [39] uses the two center
94 Lennard-Jones plus quadrupole approach (2CLJQ), employing two equal LJ
95 sites and a point quadrupole in the center of mass. The model of Merker et
96 al. [40] employs two equal LJ sites for the oxygen atoms and a different LJ

97 site for the central carbon atom, while also describing the electrostatic inter-
98 actions with a point quadrupole in the center of mass. This model is denoted
99 as 3CLJQ in the present work. Both models show almost equal performance
100 in the VLE of pure CO₂ and have also been applied successfully in studies
101 of mixtures [20, 22–24].

102 Inspired by the general idea of the CO₂-N₂O analogy, two new molecular
103 models for N₂O were developed in the present work. They follow the same
104 modeling approaches as their CO₂ counterparts: One model is of the 2CLJQ
105 type, and the other model is of the 3CLJQ type. In the latter one, the
106 LJ sites for both nitrogen atoms are equal, while the LJ site for oxygen is
107 different from these two. In both cases, the point quadrupole is located in
108 the center of mass. The structure of the two new models and that of the
109 literature models is shown schematically in Fig. 1.

110 Both models were optimized with respect to the VLE of pure N₂O in the
111 temperature range $201 \leq T / \text{K} \leq 296$, corresponding to $0.65 \leq T / T_c \leq 0.95$.

112 Two objectives were considered: the mean relative deviations in the vapor
113 pressure and in the saturated liquid density. As a reference, the equation
114 of state (EOS) of Lemmon and Span [14] was used. Correlations of VLE
115 properties of the 2CLJQ model are available as functions of the four model

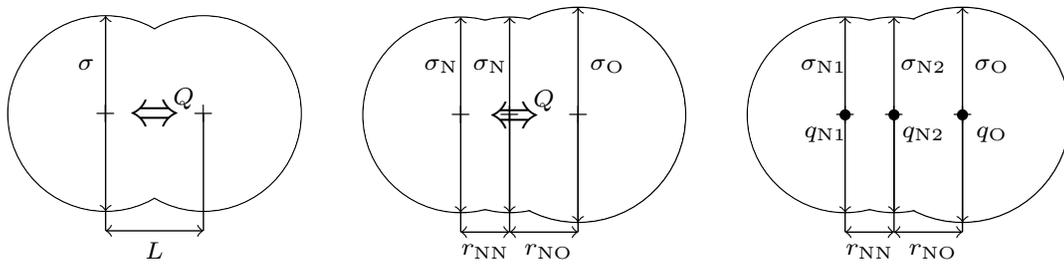


Figure 1: Structure of molecular models for nitrous oxide, schematically. Left: 2CLJQ model developed in this work. Middle: 3CLJQ model developed in this work. Right: models from the literature [27–29]. LJ sites are shown by crosses, double arrows indicate the locations of point quadrupoles, dots show partial charges. For the nitrous oxide model of Costa Gomes et al. [27], the LJ sites for N1 and N2 are equal, whereas they are different in the models of Hansen et al. [28] and Lachet et al. [29]. Note that these sketches are not in actual scale in order to emphasize the differences between the models.

116 parameters [41]. Therefore, the parametrization of 2CLJQ models can be
117 carried out efficiently, even when advanced techniques like the Pareto ap-
118 proach by Stöbener et al. are used [42]. This was done here and the Pareto
119 set of a 2CLJQ model of N₂O was determined, using the two objective func-
120 tions given above. Furthermore, one model in the Pareto knee was selected,
121 which is a good compromise between the two objectives. Molecular simula-
122 tions were then carried out to check the model quality, and a slight further
123 improvement could be achieved by the reduced units method of Merker et
124 al. [43].

125 For the 3CLJQ model, a conventional parameter optimization using a New-
126 ton scheme was performed as proposed by Eckl et al. [44] with respect to
127 the saturated liquid density and the vapor pressure. As in the 2CLJQ case,
128 further fine-tuning of the model using the reduced units method by Merker et
129 al. [43] was accomplished. The parameters of the two new molecular models
130 for N₂O are presented in Table 1.

131 *2.2. Simulation Methods*

132 The VLE of pure N₂O was computed with the Grand Equilibrium method
133 of Vrabec and Hasse [45]. To assess the predictive capability of the models,
134 the surface tension of pure N₂O was computed in MD simulations using the

Table 1: Parameters of the new molecular models for nitrous oxide. For the 3CLJQ model, both LJ sites representing nitrogen are identical. The point quadrupole is located in the center of mass for both models.

| 2CLJQ | | 3CLJQ | |
|------------------------------------|----------|---|---------|
| $\sigma / \text{\AA}$ | 3.0503 | $\sigma_{\text{N}} / \text{\AA}$ | 3.0802 |
| $\epsilon/k_{\text{B}} / \text{K}$ | 133.7920 | $\epsilon_{\text{N}}/k_{\text{B}} / \text{K}$ | 46.4285 |
| | | $\sigma_{\text{O}} / \text{\AA}$ | 3.0612 |
| | | $\epsilon_{\text{O}}/k_{\text{B}} / \text{K}$ | 88.2224 |
| $L / \text{\AA}$ | 2.2688 | $r_{\text{NN}} / \text{\AA}$ | 1.1316 |
| | | $r_{\text{NO}} / \text{\AA}$ | 1.1877 |
| $Q / \text{D}\text{\AA}$ | 3.6810 | $Q / \text{D}\text{\AA}$ | 3.7657 |

135 methods described by Werth et al. [46]. The Henry’s law constant of a solute
136 i (either CO₂ or N₂O) in the solvent *solv* (either water, ethanol, or a mixture
137 of both) was obtained by sampling the chemical potential of the solute i at
138 infinite dilution μ_i^∞ and using the relation [47]

$$H_{i,solv}(T) = \rho_{solv}RT \exp(\mu_i^\infty/k_B T), \quad (5)$$

139 where k_B is Boltzmann’s constant, R is the universal gas constant, and ρ_{solv}
140 is the density of the solvent, which can be computed on the fly. The sta-
141 tistical uncertainty of Henry’s law constant was estimated by evaluating the
142 standard deviation of μ_i^∞ with the block averaging method of Flyvbjerg and
143 Petersen [48]. The upper and lower limits for Henry’s law constant were
144 obtained by computing it from μ_i^∞ plus or minus three standard deviations.
145 All simulations of the present study were carried out with the molecular sim-
146 ulation programs *ms2* [49] and, for the simulation of the surface tension, *ls1*
147 *mardyn* [50]. Technical simulation details are given in the Appendix, and all
148 numerical simulation results of this work are tabulated in the Supplementary
149 Material.

150 **3. Results and Discussion**

151 *3.1. Molecular Models for Nitrous Oxide*

152 The VLE of the two new molecular models for N₂O is compared to the
153 EOS by Lemmon and Span [14] in Fig. 2.

154 Both models perform almost equally well for the VLE of pure N₂O. The
155 critical properties were derived from the simulation results using the proce-
156 dure proposed by Lotfi et al. [51] and agree favorably with the EOS critical
157 properties, cf. Table 2. For the comparison of the performance of the two
158 new models for N₂O with existing models from the literature, deviation plots
159 of the VLE properties are shown in Fig. 3, and the mean deviations from
160 the EOS data are presented in Table 3.

161 For the liquid and vapor densities, the two new models perform equally
162 well as the best literature model, which is that of Hansen et al. [28]. For
163 the vapor pressure, the two new models yield better results than the existing
164 ones. For the enthalpy of vaporization, the two new models are similar to
165 the model of Hansen et al. [28], whereas the Lachet et al. [29] model yields
166 the best results.

167 Additionally, the Pareto front obtained for the 2CLJQ model is shown in
168 Fig. 4 together with the deviations in ρ_l and p^s of all N₂O models.

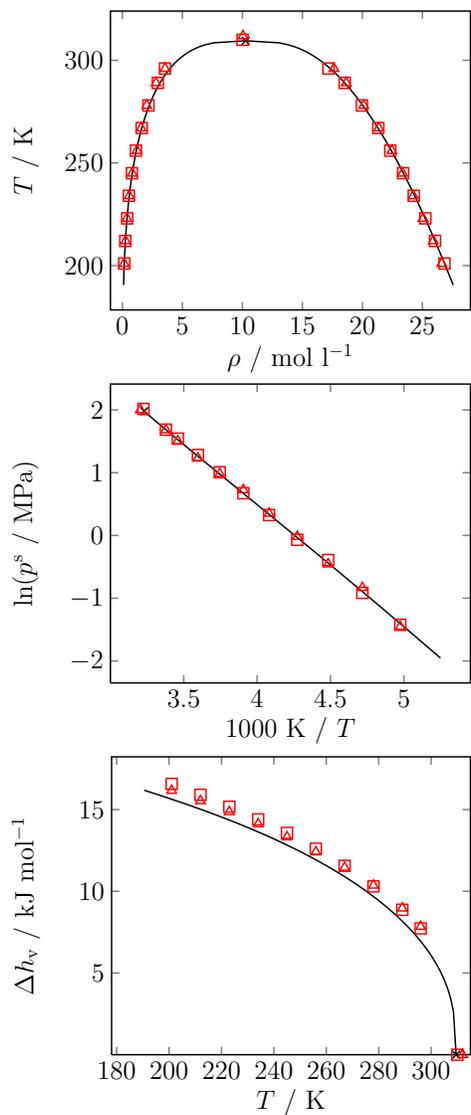


Figure 2: Vapor-liquid equilibrium of the two new molecular models for nitrous oxide. Simulation results for the 2CLJQ model (\square) and the 3CLJQ model (\triangle) are compared to the equation of state by Lemmon and Span [14] (—, the cross represents the critical point). Top panel: Saturated liquid and vapor densities. Middle panel: vapor pressure. Bottom panel: enthalpy of vaporization. Statistical uncertainties are within symbol size.

Table 2: Critical properties of N₂O as predicted by various molecular models compared to critical properties of the EOS of Lemmon and Span [14].

| Model | T_c / K | p_c / MPa | $\rho_c / \text{mol l}^{-1}$ |
|-------------------------|------------------|--------------------|------------------------------|
| EOS [14] | 309.5 | 7.25 | 10.27 |
| Costa Gomes et al. [27] | 323.7 | 7.69 | 9.79 |
| Hansen et al. [28] | 308.8 | 7.61 | 10.17 |
| Lachet et al. [29] | 304.7 | 7.47 | 10.25 |
| 2CLJQ, this work | 310.0 | 7.52 | 10.01 |
| 3CLJQ, this work | 311.8 | 7.38 | 10.05 |

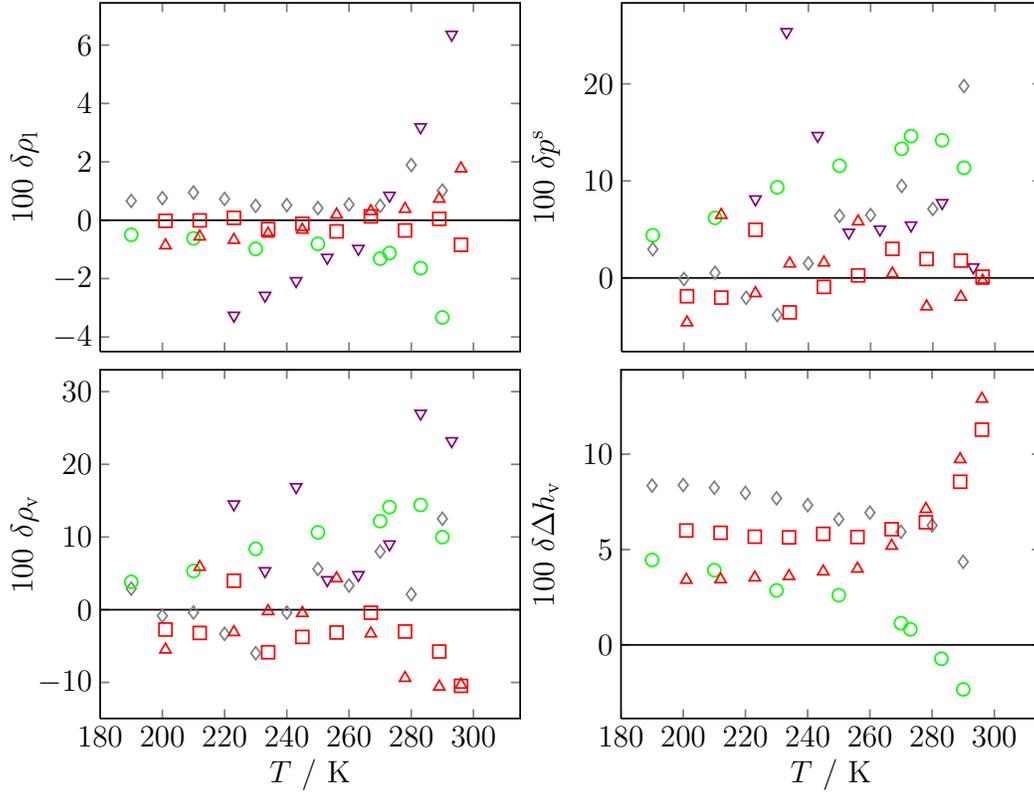


Figure 3: Relative deviations between simulation results and the EOS by Lemmon and Span [14], defined as $\delta z = (z_{\text{Sim}} - z_{\text{EOS}})/z_{\text{EOS}}$ for the VLE of nitrous oxide. Left panel: saturated liquid and vapor densities, right panel: vapor pressure and enthalpy of vaporization. Symbols correspond to different molecular models for nitrous oxide: this work 2CLJQ (\square), this work 3CLJQ (\triangle), Costa Gomes et al. [27] (∇), Hansen et al. [28] (\diamond), Lachet et al. [29] (\circ). The results for the molecular models from the literature were taken from the original publications. No data for the enthalpy of vaporization was given by Costa Gomes et al. [27]. Statistical uncertainties are not shown for the sake of clarity.

Table 3: Mean relative deviations between simulation results and the EOS by Lemmon and Span [14], defined as $\delta z = |(z_{\text{Sim}} - z_{\text{EOS}})/z_{\text{EOS}}|$ for VLE properties of nitrous oxide. The results for the molecular models from the literature were taken from the original publications. No data for the enthalpy of vaporization was given by Costa Gomes et al. [27].

| | Costa Gomes et al. [27] | Hansen et al. [28] | Lachet et al. [29] | 2CLJQ | 3CLJQ |
|--------------|-------------------------|--------------------|--------------------|-------|-------|
| ρ_l | 2.6% | 0.8% | 1.3% | 0.2% | 0.6% |
| ρ_v | 13.1% | 4.1% | 9.9% | 4.2% | 5.3% |
| p^s | 9.0% | 5.5% | 10.6% | 2.2% | 2.7% |
| Δh_v | — | 7.1% | 2.4% | 6.7% | 5.7% |

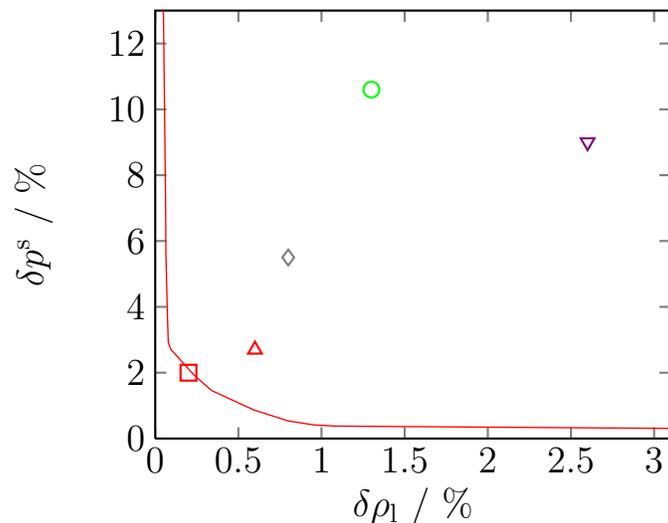


Figure 4: Pareto front for 2CLJQ models for nitrous oxide (–) and deviations of molecular models from the EOS by Lemmon and Span [14]. Symbols correspond to different molecular models for nitrous oxide: this work 2CLJQ (\square), this work 3CLJQ (\triangle), Costa Gomes et al. [27] (∇), Hansen et al. [28] (\diamond), Lachet et al. [29] (\circ).

169 The chosen 2CLJQ model lies in the region of the Pareto knee on the front.
 170 The 3CLJQ model, although more complex, does not attain the quality of
 171 the 2CLJQ model. This is not astonishing because the 3CLJQ model is
 172 asymmetric, whereas real N_2O is almost symmetric due to its mesomery [12].
 173 Even the best literature model is clearly worse than the models developed
 174 here regarding the deviations in ρ_1 and p^s . It is remarkable that the 2CLJQ
 175 model (4 parameters) outperforms the 3CLJQ model (7 parameters) and the
 176 literature models employing partial charges (8 parameters for the model of

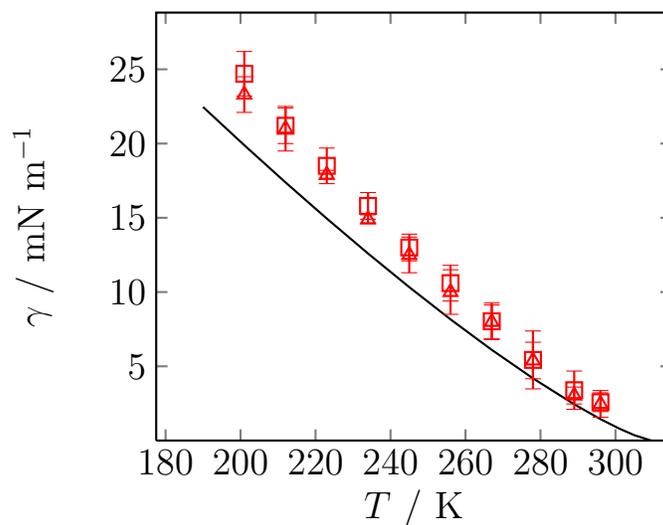


Figure 5: Surface tension of the two new molecular models for nitrous oxide. Simulation results for the 2CLJQ model (\square) and the 3CLJQ model (\triangle) are compared to the DIPPR correlation [52] (-).

177 Costa Gomes et al. [27], and 10 parameters for the other two literature models
 178 [28, 29]), even though it is conceptually much simpler and computationally
 179 more efficient.

180 As a further test of the quality of the two new molecular models for N_2O , the
 181 vapor-liquid surface tension was computed for both models. In Fig. 5, the
 182 simulation results are compared to a correlation of experimental data from
 183 the DIPPR database [52].

184 Both models overestimate the surface tension of pure N_2O . However, the
 185 trend is captured accurately and the deviations are less pronounced for higher

186 temperatures. On average, the surface tension is overestimated by about
187 25 % (2CLJQ) and 20 % (3CLJQ), which are typical values for molecular
188 models of real fluids which were parametrized to VLE data [53].

189 In summary, the two new molecular models for N₂O capture the pure fluid
190 properties well. In the following section, it is shown that also reliable data
191 on mixtures can be obtained with these models.

192 3.2. Assessment of the Analogy

193 In order to assess the model performance – not only of the solutes but
194 also of the solvents – simulations of Henry’s law constants of both gases in
195 the pure solvents water and ethanol were carried out over a wide range of
196 temperatures. Fig. 6 shows the results for pure water as the solvent, modeled
197 by TIP4P/2005 [33]. It is combined with all four solute models: the 2CLJQ
198 CO₂ model by Vrabec et al. [39], the 3CLJQ CO₂ model by Merker et al.
199 [40], and the two new models for N₂O from the present work.

200 As the predictions using the original Lorentz-Berthelot combining rules (i.e.
201 $\xi_{ij} = 1$) lead to an important overestimation of Henry’s law constant in all
202 cases, a binary interaction parameter ξ_{ij} was adjusted, cf Eq. (4). It was
203 adjusted to yield the correct Henry’s law constant of the respective solute
204 model in TIP4P/2005 water at a temperature of 313.15 K, using the cor-

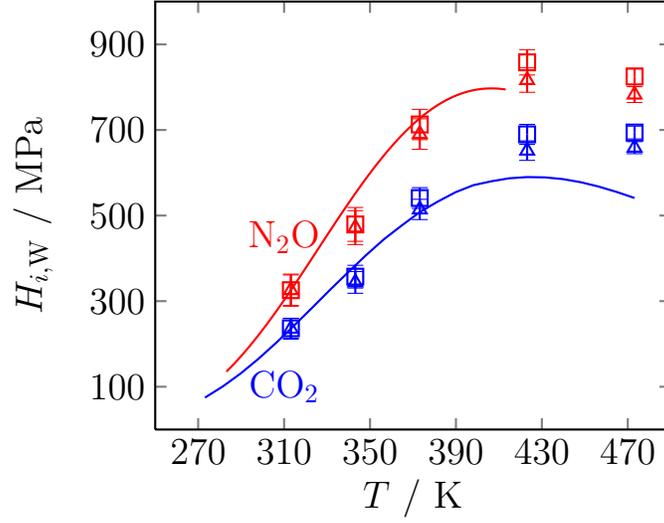


Figure 6: Henry's law constant of N₂O and CO₂ in pure water as a function of the temperature. Simulation results for N₂O (□ 2CLJQ this work, △ 3CLJQ this work) and CO₂ (□ 2CLJQ Vrabec et al. [39], △ 3CLJQ Merker et al. [40]) are compared to correlations of experimental data for N₂O of Penttilä et al. [6] (—) and for CO₂ of Rumpf and Maurer [5] (—). Binary interaction parameters $\xi_{i,W}$ were fit to $H_{i,W}(T = 313.15 \text{ K})$ given by the correlations.

Table 4: Binary interaction parameters adjusted in this work to the Henry’s law constant of N₂O and CO₂ in pure water at $T = 313.15$ K.

| Solute model | $\xi_{i,\text{TIP4P}/2005}$ |
|------------------------------|-----------------------------|
| N ₂ O, 2CLJQ | 1.0466 |
| N ₂ O, 3CLJQ | 1.0434 |
| CO ₂ , 2CLJQ [39] | 1.0748 |
| CO ₂ , 3CLJQ [40] | 1.0608 |

205 relations of Penttilä et al. [6] and Rumpf and Maurer [5] as the reference.
 206 With this adjusted value of the temperature-independent interaction param-
 207 eter ξ_{ij} , the temperature dependence of Henry’s law constant is predicted
 208 reasonably well by all four model combinations. The Henry’s law constant is
 209 only overestimated in the high temperature region, but the fact that it has a
 210 maximum is predicted correctly. The optimized interaction parameters are
 211 given in Table 4. The same systematic investigation was conducted for the
 212 solvent ethanol. The Henry’s law constants of all four solute models together
 213 with the ethanol model of Schnabel et al. [20] are presented in Fig. 7, where

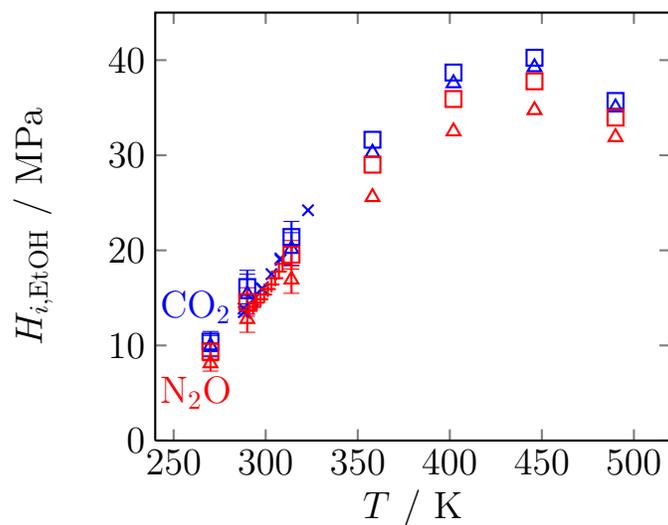


Figure 7: Henry's law constant of N₂O and CO₂ in pure ethanol as a function of the temperature. Simulation results for N₂O (□ 2CLJQ this work, △ 3CLJQ this work) and CO₂ (□ 2CLJQ Vrabec et al. [39], △ 3CLJQ Merker et al. [40]) are compared to experimental data for N₂O (+) [15] and for CO₂ (×) [16, 17]. Simulation uncertainties are only shown explicitly where they exceed the symbol size.

214 they are compared to a compilation of experimental data points from the
 215 literature.

216 In the case of ethanol, as opposed to water, the predictions of all the model
 217 combinations fit well with the experimental data in the narrow temperature
 218 range in which such data are available for a comparison. Therefore, the orig-
 219 inal Lorentz-Berthelot combining rules ($\xi_{ij} = 1$) were retained. The system
 220 '2CLJQ CO₂ + ethanol' has already been considered by Schnabel et al. [20],

221 who adjusted a binary interaction parameter of $\xi_{\text{CO}_2, \text{EtOH}} = 0.992$, which is
222 very close to 1. Here, we prefer to rely on the predictive capability of the
223 models and therefore refrain from making this slight adjustment.

224 Fig. 7 shows that the differences between the two studied CO_2 models regard-
225 ing the Henry's law constants in ethanol are small. For the two N_2O models,
226 somewhat higher differences are observed at high temperatures. The results
227 discussed above show that, where experimental data are available, the differ-
228 ences between the alternative models are small. However, for N_2O the 2CLJQ
229 and for CO_2 the 3CLJQ model are slightly better than their corresponding
230 alternatives. Therefore, the following investigation of mixed solvents is re-
231 stricted to the two solute models 2CLJQ N_2O and 3CLJQ CO_2 . Preliminary
232 simulations which we have carried out with the other solute models and which
233 are not documented here show that this choice has no influence on the main
234 conclusion from the study.

235 On this basis, 'water + ethanol' mixed solvents were considered at different
236 compositions and $T = 323.15$ K. The results are shown in Fig. 8.

237 For the solute CO_2 , experimental data are available in the mixed solvent
238 from Dalmolin et al. [17] and the model predictions are in good agreement
239 with these data. No experimental data on the solubility of N_2O in the mixed

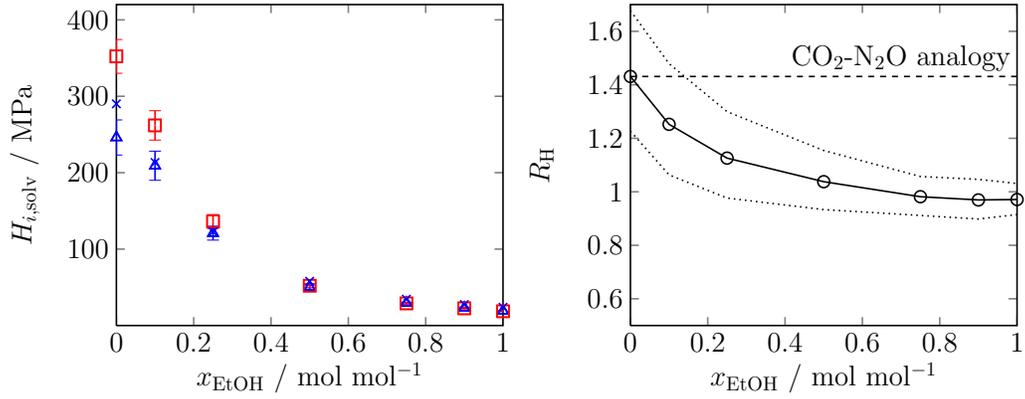


Figure 8: Henry's law constant of N₂O and CO₂ over the gas-free solvent composition in water + ethanol mixed solvents at $T = 323.15$ K. Left panel: Simulation results for the 2CLJQ N₂O model (\square) and the 3CLJQ CO₂ model [40] (\triangle) are compared to experimental data for CO₂ of Dalmolin et al. [17] (\times). Right panel: Ratio of Henry's law constants $R_H = H_{N_2O,solv}/H_{CO_2,solv}$ from molecular simulation (\circ), the solid line is a guide to the eye. The dotted lines show the confidence band estimated from the uncertainties in the simulated Henry's law constants. The dashed line shows the constant ratio assumed in the CO₂-N₂O analogy.

240 solvent are available. This gap is closed by the present molecular simulations.
241 The right panel of Fig. 8 shows the ratio of Henry’s law constants R_H , cf.
242 Eq. (1), as a function of the solvent composition. Adding ethanol to water
243 results in a sharp drop of the ratio R_H . Only in the ethanol-rich region, this
244 ratio R_H is approximately constant. Clearly, the $\text{CO}_2\text{-N}_2\text{O}$ analogy is not
245 valid for the investigated solvent mixture water + ethanol. This is confirmed
246 also by the experimental Henry’s law constant ratios in the pure solvents
247 alone: The ratio is found to be $R_H \approx 1.4$ in water, whereas it is $R_H \approx 1.0$
248 in ethanol. The present simulations show that the analogy already breaks
249 down for small mole fractions of ethanol in water, sooner than one would
250 have expected based on these data alone.

251 4. Conclusion

252 Two new molecular models for nitrous oxide, a 2CLJQ and a 3CLJQ
253 model, were developed. Both models describe the VLE of nitrous oxide
254 equally well, and are in better agreement with the reference equation of state
255 than any literature model available to date. The two models also provide
256 reliable results for the Henry’s law constants of nitrous oxide in the two
257 pure solvents water and ethanol. The same was found for the 2CLJQ model

258 of Vrabec et al. [39] and the 3CLJQ model of Merker et al. [40] for carbon
259 dioxide. An adjustment of the binary interaction gas-solvent is only necessary
260 for water, whereas gas solubilities in ethanol are predicted quantitatively
261 without considering binary data.

262 The successful study of the solubility of CO₂ and N₂O in the pure solvents was
263 the basis for an investigation of ternary systems. Using molecular simulation,
264 it was shown that the CO₂-N₂O analogy is invalid for the solvent mixture
265 water + ethanol. The methodology presented here can be used to study
266 other systems relevant for carbon dioxide removal from gas streams in future
267 work. The high quality of the results of the molecular simulations found in
268 the present work indicates that physical gas solubilities in reactive solvents
269 can be predicted reliably by molecular models.

270 **Acknowledgments**

271 The present work was conducted under the auspices of the Boltzmann-
272 Zuse Society of Computational Molecular Engineering (BZS) and the sim-
273 ulations were carried out on the Regional University Computing Center
274 Kaiserslautern (RHRK) under the grant TUKL-MSWS, the High Perfor-
275 mance Computing Center Stuttgart (HLRS) under the grant MMHBF as

276 well as the Leibniz Supercomputing Centre (LRZ) under the grant SPAR-
277 LAMPE (pr48te). The authors thank Stephan Deublein and Steffen Reiser
278 for fruitful discussions.

279 **Funding**

280 This work was supported by the Reinhart Koselleck Program (HA1993/15-
281 1) of the German Research Foundation (DFG).

282 **Appendix A. Simulation Details**

283 In all simulations of the present study, electrostatic long-range interac-
284 tions were calculated using the reaction field method [54] and the tempera-
285 ture was kept constant using a velocity scaling thermostat. In all molecular
286 dynamics (MD) simulations, the time step was 1.2 fs.

287 *Appendix A.1. VLE of Pure Nitrous Oxide*

288 For the VLE of pure nitrous oxide, the Grand Equilibrium method by
289 Vrabec and Hasse [45] was used. First, the chemical potential of pure nitrous
290 oxide in the liquid phase was sampled with Widom’s test particle insertion
291 in MD simulations in the NpT ensemble, using 1372 particles and Berend-
292 sen’s barostat. The cutoff radius was 19.5 Å. The fluid was equilibrated for

293 25000 time steps, before sampling was carried out for 200000 time steps. Up
294 to 5000 test particles were inserted every time step. Then, the vapor phase
295 was simulated in a pseudo- μVT ensemble Monte Carlo (MC) simulation, in
296 which equilibration and production took 20000 and 50000 MC loops, respec-
297 tively. Each loop consisted of $N_{\text{NDF}}/3$ steps, where N_{NDF} indicates the total
298 number of mechanical degrees of freedom of the system, plus two insertion
299 and deletion attempts. The cutoff radius was equal to half the box length.

300 *Appendix A.2. Henry's Law Constant*

301 Henry's law constant was sampled according to Eq. (5) of the main text.
302 MD simulations were carried out with 1000 solvent molecules in the NpT
303 ensemble at pressures close to the vapor pressure of the pure solvent at the
304 respective temperature. The chemical potential of the solute was sampled
305 with Widom's test particle insertion, again with up to 5000 trial insertions
306 per time step. Equilibration and production took 50000 and 1000000 time
307 steps, respectively, and the cutoff radius was 15.5 Å.

308 *Appendix A.3. Surface Tension*

309 The surface tension of pure N_2O was computed in MD simulations with
310 the approach of Werth et al. [46]. The simulations were carried out in the

311 *NVT* ensemble, and equilibration and production took 500000 and 2500000
312 time steps, respectively. The cutoff radius was 19.5 Å.

313 **Nomenclature**

314 *Abbreviations*

| | |
|-------|--|
| 2CLJQ | Two center Lennard-Jones plus quadrupole |
| 3CLJQ | Three center Lennard-Jones plus quadrupole |
| EOS | Equation of state |
| EtOH | Ethanol |
| LJ | Lennard-Jones |
| MC | Monte Carlo |
| MD | Molecular dynamics |
| VLE | Vapor-liquid equilibrium |
| W | Water |

315 *Symbols*

| | |
|--------------|------------------------------|
| γ | Vapor-liquid surface tension |
| ϵ | LJ energy parameter |
| ϵ_0 | Vacuum permittivity |
| f_1, f_2 | Angle-dependent functions |

| | |
|--------------|---|
| Δh_v | Enthalpy of vaporization |
| $H_{i,j}$ | Henry's law constant of solute i in solvent j |
| k_B | Boltzmann's constant |
| L | Elongation |
| μ_i | Chemical potential of component i |
| n | Number of sites |
| N | Number of molecules |
| ω | Angle |
| p | Pressure |
| q | Point charge |
| Q | Quadrupole moment |
| ρ | Density |
| r | Distance |
| R | Universal gas constant |
| R_H | Ratio of Henry's law constants |
| σ | LJ size parameter |
| T | Temperature |
| U | Potential |
| V | Volume |

ξ_{ij} Binary interaction parameter

x Mole fraction

316 *Subscripts and Superscripts*

a, b, c, d Site index

c Critical

e Electrostatic

i, j Component / molecule index

l Saturated liquid

N Nitrogen

O Oxygen

s Saturated

$solv$ Solvent

v Saturated vapor

∞ At infinite dilution

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