The influence of the liquid slab thickness on the planar vapor-liquid interfacial tension

Stephan Werth, Sergey Lishchuk, Martin Horsch,¹ Hans Hasse

Laboratory of Engineering Thermodynamics, Department of Mechanical and Process Engineering, University of Kaiserslautern, Erwin-Schrödinger-Str. 44, 67663 Kaiserslautern, Germany

Abstract

One of the long standing challenges in molecular simulation is the description of interfaces. On the molecular length scale, finite size effects significantly influence the properties of the interface such as its interfacial tension, which can be reliably investigated by molecular dynamics simulation of planar vaporliquid interfaces. For the Lennard-Jones fluid, finite size effects are examined here by varying the thickness of the liquid slab. It is found that the surface tension and density in the center of the liquid region decreases significantly for thin slabs. The influence of the slab thickness on both the liquid density and the surface tension is found to scale with $1/S^3$ in terms of the slab thickness S, and a linear correlation between both effects is obtained. The results corroborate the analysis of Malijevský and Jackson, J. Phys.: Cond. Mat. 24: 464121 (2012), who recently detected an analogous effect for the surface tension of liquid nanodroplets.

Keywords: Surface tension, long range correction, finite size effects

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1 1. Introduction

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² Molecular simulation is a well-established approach for the analysis of fluid ³ interfaces and their molecular structure. Much work has been dedicated ⁴ to the interfacial tension.¹⁻⁴ For a fluid interface, there are (at least) three ⁵ different aspects in which its size can be varied, each of which may affect the ⁶ interfacial tension:

- curvature effects, depending on the local characteristic radii of curva ture
- capillary wave effects, depending on the range of wavelengths permitted
 by the morphology and size of the interface
 - confinement effects, which arise due to spatial restrictions imposed on a fluid phase by one or several interfaces or walls

According to the Tolman⁵ approach, the interfacial tension of a nanodroplet 13 deviates from that of a planar interface due to its extremely curved shape.^{6–10} 14 However, it should be noted that all three phenomena are present when the 15 size of a droplet is varied: Smaller droplets have a higher curvature, a smaller 16 range of capillary wavelengths, and a more significant deviation from bulk-17 like behaviour due to confinement. In addition to curvature, the other effects 18 might therefore also have a significant influence on the formation of droplets 19 in a supersaturated vapor, where the size of the critical nucleus and the 20

 $^{^{1}\}mathrm{Corresponding}$ author: Martin Horsch, martin.horsch@mv.uni-kl.de, +49 631 205 4028.

nucleation rate are of major interest.¹¹⁻¹⁵ A similar case is cavitation, where
bubbles emerge in a liquid phase.¹⁶⁻¹⁸

Spherically curved interfaces of droplets were simulated for the first time 23 by molecular simulation in the early 1970s.^{19,20} Nonetheless, while curvature 24 effects are relatively well-studied, there are only astonishingly few investi-25 gations related to the other size effects, which are also present in the case 26 of planar interfaces. Several previous works address the influence of small 27 simulation volumes,^{21–24} which is usually discussed in terms of capillary wave 28 effects.^{24–28} The present study considers the influence of the liquid slab thick-29 ness, i.e. of confinement by two parallel planar vapor-liquid interfaces which 30 are close to each other. This effect was previously investigated by Weng et 31 al.²⁹ who, however, did not find a systematic correlation. 32

The computation of interfacial properties is always done in a single sim-33 ulation volume containing both phases, the liquid and the vapor phase, sep-34 arated by the interface. For the calculation of the bulk properties there are 35 many other methods, like Grand Equilibrium,³⁰ NpT plus test particle,³¹ 36 or the Gibbs ensemble method.³² The surface tension can be computed for 37 example via the virial route or the surface free energy.^{2,33–37} The virial route 38 is directly related to the common approach for calculating the pressure in a 39 molecular simulation.^{22,23,38,39} It is known that the pressure in dependence 40 of the density exhibits a van der Waals loop in the two phase region.^{40,41} 41

At interfaces, the long range contribution to the interaction potential plays an important role for all thermodynamic properties.^{42–44} Nonetheless, there are also simulations applying a truncated and shifted potential, which neglects the whole long range contribution.^{16,45–47} When dealing with a ho⁴⁶ mogeneous system, long range corrections are only needed for energy and
⁴⁷ pressure, ^{48,49} while in an inhomogeneous configuration, also the dynamics of
⁴⁸ the systems needs to be appropriately corrected. ⁵⁰⁻⁵²

In the present work, the influence of the liquid slab thickness on thermodynamic properties is discussed. A suitable long range correction is used to obtain accurate and validated results.

⁵² 2. Simulation method

⁵³ In this study, the Lennard-Jones potential (LJ)

$$u_{ij} = 4\epsilon \left[\left(\frac{\sigma}{r_{ij}} \right)^{12} - \left(\frac{\sigma}{r_{ij}} \right)^6 \right]$$
(1)

is employed, where ϵ and σ are the energy and size parameters and r_{ij} is the distance between the two particles *i* and *j*. The standard LJ parameters $\epsilon = 1$ and $\sigma = 1$ are used, yielding a reduced LJ potential.

As usual, the potential was truncated in order to reduce the computing 57 time. A cutoff radius of $r_c = 3$ was used for the present simulations. To 58 diminish the error made by this assumption, a bin based tail correction was 59 applied to the simulation.^{50,53} Thereby, the potential energy, the forces acting 60 on the molecules, and the virial are each split into an explicitly computed 61 part and a long range correction. The calculation of the correction terms 62 was conducted every 10 time steps. It is known that this method provides 63 cutoff independent results for the LJ fluid.⁵⁰ For a discussion of the employed 64 method in full detail, the reader is referred to Janeček's work, ⁵⁰ wherein this 65 approach was first presented. 66

The interfacial tension γ is given by the difference between the diagonal components of the virial tensor $\Pi_N - \Pi_T$ or, equivalently, an integral over the differential pressure^{50,54} $p_N - p_T$

$$\gamma = \frac{1}{2A} \left(\Pi_N - \Pi_T \right) = \int_{-\infty}^{\infty} \left(p_N - p_T \right) \mathrm{d}y, \tag{2}$$

where 2*A* denotes the interfacial area of the two interfaces. The pressure calculation is based on the method proposed by Irving and Kirkwood,⁵⁵ but in contrast to their approach the pressure is not homogeneously distributed between the positions of the two particles. To speed up the simulation the pressure is divided between the bins of the involved particles. It should be noted that this modification has a slight effect on the localized pressure tensor but leads to the same outcome for the overall surface tension.²³

⁷⁷ Molecular dynamics (MD) simulations were conducted in the canonical ⁷⁸ ensemble with $N = 16\ 000$ particles. The equimolar thickness of the li-⁷⁹ quid slab S was varied between 12 and the minimum stable configuration. ⁸⁰ The equimolar thickness was determined using the saturated densities ρ' and ⁸¹ ρ'' for the given temperature, the simulation volume V, and the number of ⁸² particles N

$$S = \frac{N - \rho'' V}{(\rho' - \rho'')A},\tag{3}$$

i.e. S only depends on the boundary conditions applied to the molecular simulation within the canonical ensemble, not on the outcome of the simulation, and it does not vary over simulation time. The temperature was kept constant by a velocity scaling thermostat. All simulations were performed in a parallelepiped box. The elongation of the simulation volume in y direction, i.e. normal to the interface, was $\ell_y = 50$.

For the simulation of a reference case the number of particles N was 89 increased to 300 000, the elongation in y direction was $\ell_y = 100$ and a slab 90 thickness of S = 40 was used. The temperature T was varied between 0.7 and 91 1.25, i.e. from the triple point temperature to 95% of the critical temperature. 92 The simulations were carried out using the ls1 molecular dynamics code.⁵⁶ 93 The equation of motion was solved by a leapfrog integrator.⁵⁷ A time step of 94 $\Delta t = 0.002$ was used. The equilibration was conducted for at least 120 000 95 time steps, while the production ran for 840 000 time steps. The statistical 96 errors given in the present study are equal to 3 times the standard deviation 97 of 7 block averages, each over 120 000 time steps. 98

⁹⁹ A further series of simulations was conducted to validate whether rela-¹⁰⁰ tively small cutoff radii are permissible when Janeček's cutoff correction is ¹⁰¹ employed. The simulation results support this conclusion, cf. Fig. 1.

[Figure 1 about here.]

103 3. Results

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[Figure 3 about here.]

As described above, a series of simulations was carried out using a large liquid slab of S = 40 in order to approximate bulk phase behavior. The resulting values γ_{∞} and ρ'_{∞} are used as a reference for the further simulations. The resulting surface tension is shown in Fig. 2. The regression

[Figure 2 about here.]

$$\gamma = 2.94 \left(1 - \frac{T}{T_c} \right)^{1.23} \tag{4}$$

is obtained, with $T_c = 1.3126$ according to Pérez Pellitero et al.⁵⁸ The type of correlation is the same as proposed by Vrabec et al.⁴⁶ for the truncated and shifted LJ potential, and their exponent (i.e. 1.21) is also very similar to the present one.

Moreover, simulations were also performed for smaller slab thicknesses 114 $(S \leq 12)$. Thereby, MD runs were conducted with successively smaller values 115 of S, until a minimum stable thickness was reached for the given temperature. 116 In Fig. 3, the density and the corresponding differential pressure profile 117 for T = 0.7 is plotted over the y coordinate. It can be seen that the density 118 in the center of the slab for S = 12 and S = 7 almost matches the bulk liquid 119 density at saturation, which is also plotted in Fig. 3 as reference. It is slightly 120 smaller for S = 4.3. In the differential pressure the difference between the 12 three simulations is more significant. The differential pressure in the center of 122 the slab (y = 0) almost reaches the zero line for S = 12, while for S = 7 and 123 S = 4.3 the pressure tensor is anisotropic throughout the liquid slab. The 124 differential pressure can be seen as an indicator for the fluid to be isotropic 125 or homogeneous, i.e. not influenced by the two interfaces. For S larger than 126 12, the differential pressure fluctuates around zero in the center of the liquid 127 slab for T = 0.7. 128

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[Figure 4 about here.]

In Fig. 4, the plot of the differential pressure over the density is shown, which is obtained from the data displayed in Fig. 3. Additionally results for T = 1.0 are shown. The plot in Fig. 4 exhibits van der Waals loops in all cases. The red squares and green stars correspond to a large liquid slab, while the blue circles and brown triangles show the result of the smallest
stable liquid slab. Like in Fig. 3, it is obvious that the differential pressure
does not reach zero in the latter case.

The resulting surface tensions are shown in Tabs. 1 and 2. For all temperatures the surface tension decreases when the liquid slab thickness decreases.

139 [Table 1 about here.]

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[Table 2 about here.]

The differential pressure in the center of the liquid slab, i.e. the minimum 141 differential pressure, is shown in Tabs. 3 and 4. As already discussed, the 142 differential pressure in the center of the liquid slab increases with decreasing 143 slab thickness. The density corresponding to the minimum differential pres-144 sure, i.e. the maximum density, in the liquid slab is shown in Tabs. 5 and 6. 145 At the largest slab thickness the value agrees with the bulk properties. For 146 lower slab thicknesses the density does not reach the bulk values. The mini-147 mum stable slab thickness increases with increasing temperature. Reducing 148 the slab thickness below this point results in a rupture of the liquid phase 149 and a transition from planar to cylindrical or spherical symmetry. 150

151	[Table 3 about here.]
152	[Table 4 about here.]
153	[Table 5 about here.]
154	[Table 6 about here.]

[Figure 5 about here.]

In Fig. 5, the relative surface tension – reduced by $\gamma_{\infty}(T)$ as obtained 156 from the large slab simulations – is plotted over the slab thickness for differ-157 ent temperatures. Confinement between two planar vapor-liquid interfaces 158 reduces the surface tension, and the numerical data suggest that this effect 159 is of the order $1/S^3$. In Fig. 6, the reduced density ρ'/ρ'_{∞} is plotted over the 160 slab thickness for different temperatures. The relative density also decreases 16 upon decreasing the slab thickness and, similar to the surface tension, this 162 effect is approximately proportional to $1/S^3$ and becomes more significant at 163 high temperatures. 164

For a slab thickness S > 12, the surface tension agrees with the value for a large liquid slab, within the simulation uncertainty. For a fluid described by the LJ potential, e.g. methane,⁵⁹ this means that confinement effects are significant for slabs which are thinner than 4.5 nm. At high temperatures, the density in the center of the liquid slab deviates more significantly from the bulk value.

¹⁷¹ [Figure 6 about here.] ¹⁷² [Figure 7 about here.]

The present results suggest that the reduction of the density and the surface tension due to confinement are related effects. In Fig. 7, the respective ratios are displayed together, which discloses an approximately linear relationship. The regression

$$\frac{\rho'}{\rho_{\infty}'} \approx 0.76 \frac{\gamma}{\gamma_{\infty}} + 0.24 \tag{5}$$

¹⁷⁷ is obtained from the simulation results.

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178 4. Conclusion

In the present work, molecular simulation was applied to study the influence 179 of the slab thickness on the interfacial properties for planar vapor-liquid in-180 terfaces. The present results prove that such an effect exists for thin slabs 18 and quantifies it for the LJ fluid. The surface tension decreases with de-182 creasing slab thickness, and so does the density in the middle of the slab. 183 The differential pressure does not reach zero for liquid slabs smaller than 12, 184 which proves that under such conditions, a bulk-like region is absent. The 185 confinement effects for the surface tension and the density were found to scale 186 with $1/S^3$ in terms of the slab thickness S, so that a linear relation between 18 both effects could be obtained. 188

The present results depart from those obtained by Weng et al.²⁹ in a 189 previous study, where no systematic correlation between the slab thickness 190 and the surface tension was found. For a LJ system at T = 0.818, Weng et 19 al. detected minor fluctuations around a constant value ($\gamma = 0.78 \pm 0.02$), 192 without a clear tendency, for a range of slab thicknesses between S = 5.0193 and 9.0.²⁹ A juxtaposition with the present numerical data, cf. Tab. 1 and 194 Fig. 5, according to which varying the slab thickness to such an extent has a 19 significant influence on γ , clearly shows that there is a contradiction between 196 present simulation results and the postulate of Weng et al. that "with film 19 thickness ... surface tension values and density profiles show little varia-198 tion."²⁹ For the simulations of Weng et al.,²⁹ however, no long-range cutoff 199 correction was employed at all, and the computations were only carried out 200 over 120 000 time steps, as opposed to a million time steps for the present 20 series of simulations. Since systems with an interface relax more slowly than 202

the homogeneous bulk fluid, the extremely short simulation time could consitute a serious limitation, affecting the accuracy of the results obtained by Weng et al.²⁹ to a significant extent.

The comparison with results from a recent study of Malijevský and Jack-206 son⁶⁰ suggests that the present results on confinement by two parallel planar 20 vapor-liquid interfaces might also carry over qualitatively to confinement by 208 the opposite sides of the single spherical interface that surrounds a small 209 droplet. Therein, Malijevský and Jackson come to the conclusion that for 210 liquid drops, the size dependence of the surface tension is best described 211 by two distinct, additive terms: The conventional Tolman term, represent-212 ing curvature, which increases the surface tension (i.e. the Tolman length 213 is found to be negative), as well as "an additional curvature dependence 214 of the $1/R^3$ form" which causes an eventual decrease of the surface tension 215 "for smaller drops."⁶⁰ Furthermore, Malijevský and Jackson observe that the 216 characteristic droplet radius, below which this negative corrective term be-21 comes dominant, "increases with increasing $r_{\rm c}$ " and conjecture that "such a 218 crossover occurs when ... no 'bulk' region can be assigned inside the drop. 219 In this case even particles in the centre of the drop 'feel' the interface." 220

The present results lend further plausibility to this conjecture of Malijevský and Jackson. There could be a relation between their $1/R^3$ term and the $1/S^3$ confinement effect from the present study. According to such a hypothesis, these contributions would both represent the deviation from bulk-like behavior of the liquid phase due to confinement.

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Figure 1: Dependence of the computed surface tension γ on the employed cutoff radius $r_{\rm c}$, for the truncated and shifted LJ potential without long-range effects (LJTS) and for the full LJ potential with a long-range correction (LRC) according to Janeček.⁵⁰ The simulations were carried out in the canonical ensemble with N = 2 048 particles at T = 0.7 using a simulation box with an elongation of $\ell_y = 66$ in the direction perpendicular to the vapor-liquid interfaces.



Figure 2: Surface tension γ over the temperature for large liquid slabs. Comparison of results of different authors: (red squares) This work, (blue triangles) Janeček,⁵⁰ (red diamonds) Holcomb et al.,⁶¹ (brown circles) Potoff and Panagiotopoulos,⁶² (black circles) in't Veld et al.,⁶³ (blue stars), as well as López Lemus and Alejandre,⁶⁴; (solid line) regression from Eq. (4).



Figure 3: Density ρ (top) and differential pressure $p_N - p_T$ (bottom) over the y coordinate (i.e. the direction perpendicular to the interface). The temperature is T = 0.7. The blue dashed line corresponds to the minimum stable configuration which is S = 4.3 for this temperature, while the red dash dotted one corresponds to S = 7 and the black solid one to S = 12. The dotted line in the upper picture represents the bulk liquid density and the difference between the vertical dashed lines in the upper picture represent the equimolar slab thickness.



Figure 4: Differential pressure $p_N - p_T$ over the density for the temperature T = 0.7 and T = 1.0. The blue circles and brown triangles correspond to the minimum stable configuration for the corresponding temperature, while the red squares and green stars correspond to S = 12. The dotted line represents the zero line.



Figure 5: Reduced surface tension γ/γ_{∞} over the slab thickness S for different temperatures. The dashed lines represent the expression $\gamma/\gamma_{\infty} = 1 - a(T)/S^3$, where the temperature-dependent coefficients were adjusted to the simulation results, yielding a(0.7) = 5.8, a(0.8) = 7.9, a(0.9) = 9.3, a(1.0) = 16, a(1.1) = 28, a(1.2) = 34, and a(1.25) = 93.



Figure 6: Reduced density ρ'/ρ'_{∞} over the slab thickness S for different temperatures. The dashed lines represent the expression $\rho'/\rho'_{\infty} = 1 - b(T)/S^3$, where the temperaturedependent coefficients were adjusted to the simulation results, yielding b(0.7) = 3.0, b(0.8) = 4.4, b(0.9) = 6.3, b(1.0) = 12, b(1.1) = 27, b(1.2) = 37, and b(1.25) = 85.



Figure 7: Reduced density ρ'/ρ'_{∞} over the reduced surface tension γ/γ_{∞} for different temperatures. The dash dotted line represents the regression from Eq. (5).

T	N	ℓ_y	ℓ_{\parallel}	S	γ
0.7	300 000	100	94.2	40.0	1.150(4)
	16000	50.0	39.7	12.0	1.14(7)
	16000	50.0	45.8	9.0	1.14(6)
	16000	50.0	51.8	7.0	1.13(3)
	16000	50.0	61.1	5.0	1.10(2)
	16000	50.0	64.3	4.5	1.08(1)
	16000	50.0	65.7	4.3	1.06(2)
0.8	300 000	100	96.4	40.0	0.93(1)
	16000	50.0	40.3	12.0	0.92(6)
	16000	50.0	46.4	9.0	0.92(3)
	16000	50.0	52.3	7.0	0.91(3)
	16000	50.0	61.2	5.0	0.87(3)
	16000	50.0	63.9	4.55	0.85(2)
0.9	300 000	100	98.2	40.0	0.707(8)
	16000	50.0	40.8	12.0	0.71(5)
	16000	50.0	46.5	9.0	0.70(4)
	16000	50.0	51.9	7.0	0.69(3)
	16000	50.0	55.4	6.0	0.68(2)
	16000	50.0	58.1	5.4	0.66(3)

Table 1: Surface tension γ in dependence of the slab thickness S (low temperatures). The total elongation of the simulation box is indicated as ℓ_y in the direction perpendicular to the vapor-liquid interfaces and as ℓ_{\parallel} (= $\ell_x = \ell_z$) in the other spatial directions. The statistical error in terms of the final digit is shown in parentheses.

T	N	ℓ_y	ℓ_{\parallel}	S	γ
1.0	300 000	100	100	40.0	0.502(5)
	16000	50.0	40.1	12.0	0.50(6)
	16000	50.0	46.1	9.0	0.50(2)
	16000	50.0	50.8	7.0	0.48(5)
	16000	50.0	53.0	6.2	0.5(1)
1.1	300 000	100	102	40.0	0.310(4)
	16000	50.0	40.9	12.0	0.31(4)
	16000	50.0	45.3	9.0	0.30(3)
	16000	50.0	48.6	7.25	0.28(4)
	16000	50.0	50.2	6.5	0.26(3)
1.2	300 000	100	101	40.0	0.144(8)
	16000	50.0	38.2	12.0	0.14(5)
	16000	50.0	40.8	9.0	0.14(4)
	16000	50.0	42.4	7.5	0.13(4)
1.25	300 000	100	101	40.0	0.075(4)
	16000	50.0	37.2	12.0	0.08(5)
	16000	50.0	39.2	9.0	0.06(2)
	16000	50.0	39.9	8.0	0.06(4)

Table 2: Surface tension γ in dependence of the slab thickness S (high temperatures), cf. Tab. 1.

	T			
S	0.7	0.8	0.9	1.0
12	-0.01(6)	-0.01(4)	0.00(3)	0.00(2)
9	0.04(6)	0.04(4)	0.04(2)	0.04(5)
7	0.13(5)	0.12(3)	0.12(4)	0.11(5)
6.2	n/a	n/a	n/a	0.16(5)
6	n/a	n/a	0.18(2)	*
5.4	n/a	n/a	0.24(3)	*
5	0.34(2)	0.32(3)	*	*
4.55	n/a	0.38(6)	*	*
4.5	0.43(4)	*	*	*
4.3	0.48(3)	*	*	*

Table 3: Differential pressure $p_N - p_T$ in the center of the liquid slab in dependence on the slab thickness S (low temperatures). "n/a": data not determined; asterisks: liquid slab found to be unstable.

		T	
S	1.1	1.2	1.25
12	0.01(2)	0.00(5)	0.00(3)
9	0.05(2)	0.02(4)	0.01(3)
8	n/a	n/a	0.02(2)
7.5	n/a	0.03(3)	*
7.25	0.10(3)	*	*
6.5	0.12(2)	*	*

Table 4: Differential pressure p_N-p_T in the center of the liquid slab (high temperatures), cf. Tab. 3.

	T				
S	0.7	0.8	0.9	1.0	
40	0.8410(2)	0.7974(3)	0.7507~(6)	0.699(1)	
12	0.84(3)	0.80(2)	0.75(1)	0.70(1)	
9	0.84(1)	0.80(1)	0.75(1)	0.69(2)	
7	0.83(1)	0.79(1)	0.74(2)	0.68(2)	
6.2	n/a	n/a	n/a	0.66(5)	
6	n/a	n/a	0.73(2)	*	
5.4	n/a	n/a	0.72(1)	*	
5	0.82(1)	0.77(1)	*	*	
4.55	n/a	0.76(3)	*	*	
4.5	0.81(1)	*	*	*	
4.3	0.81(1)	*	*	*	

Table 5: Density ρ' in the center of the liquid slab in dependence on the slab thickness S (low temperatures).

		T	
S	1.1	1.2	1.25
40	0.6393(4)	0.564(2)	0.515(7)
12	0.63(3)	0.56(3)	0.49(10)
9	0.62(2)	0.54(3)	0.46(4)
8	n/a	n/a	0.43~(15)
7.5	n/a	0.51~(7)	*
7.25	0.59(3)	*	*
6.5	0.57(8)	*	*

Table 6: Density ρ' in the center of the liquid slab (high temperatures), cf. Tab. 5.