



# Nucleation processes and vapor-liquid interfaces of fluid mixtures

M. Horsch, K. Langenbach, M. Lautenschläger, K. Stübener, S. Werth, P. Neumann, N. Tchipev, H.-J. Bungartz, S. Eckelsbach, J. Vrabec, H. Hasse

Laboratory of Engineering Thermodynamics, University of Kaiserslautern,  
Scientific Computing in Computer Science, TU Munich,  
Thermodynamics and Energy Technology, University of Paderborn

Statistical Mechanics Seminar

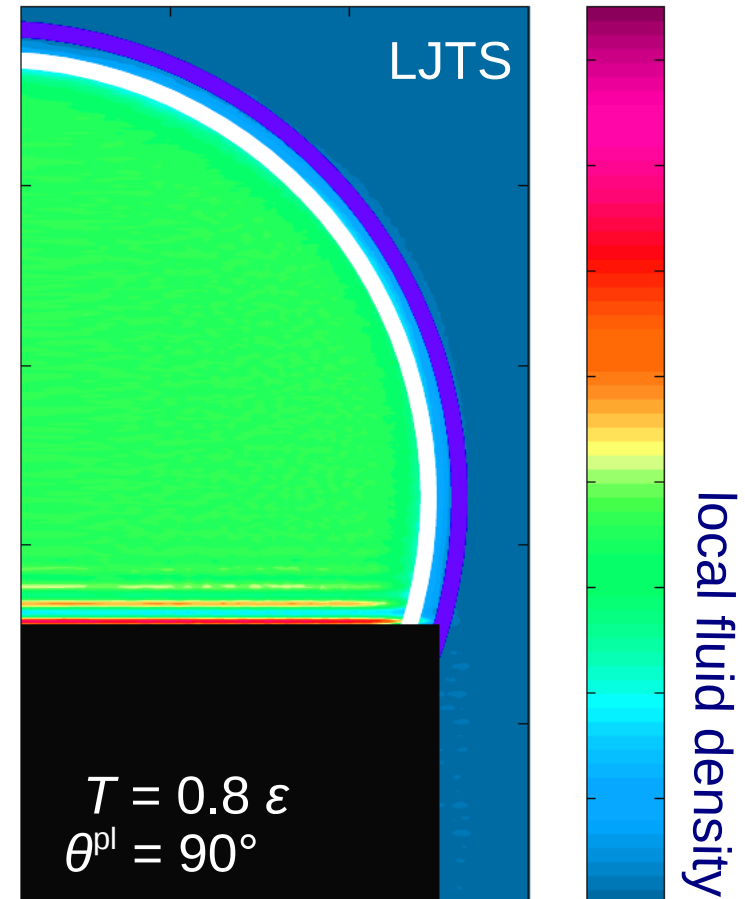
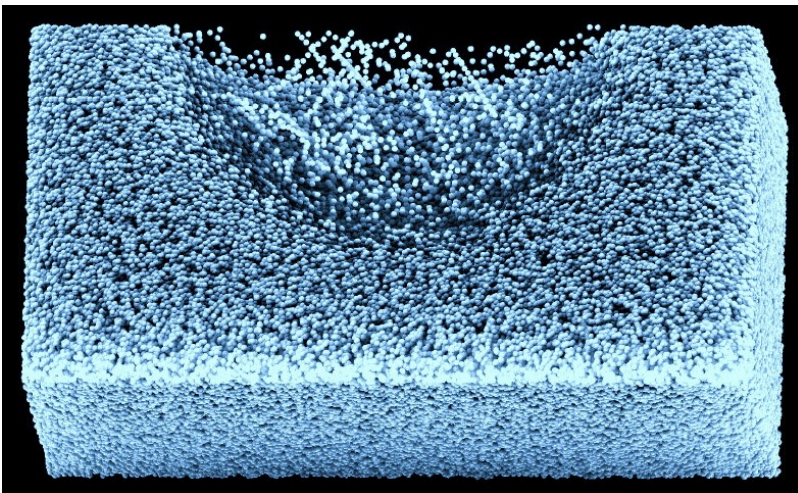
UC Berkeley, November 6, 2015



**Computational  
Molecular Engineering**

# Molecular simulation of fluids at interfaces

- Vapor-liquid surface tension
- Curved vapor-liquid interfaces
- Adsorption (fluid-fluid and fluid-solid)
- Contact angle and contact line pinning

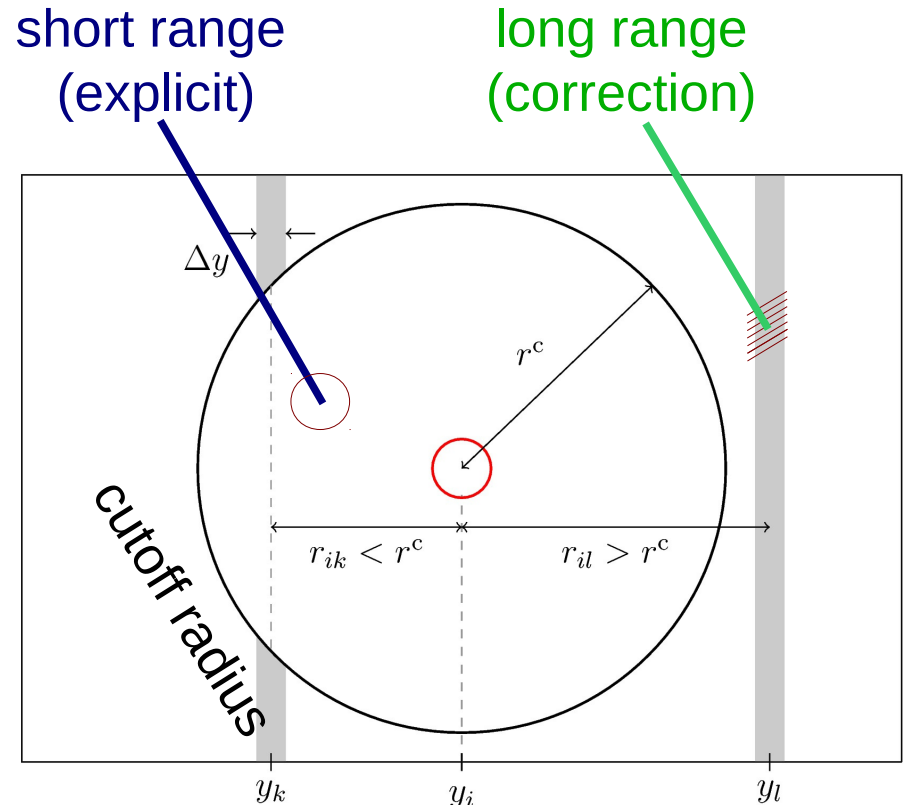
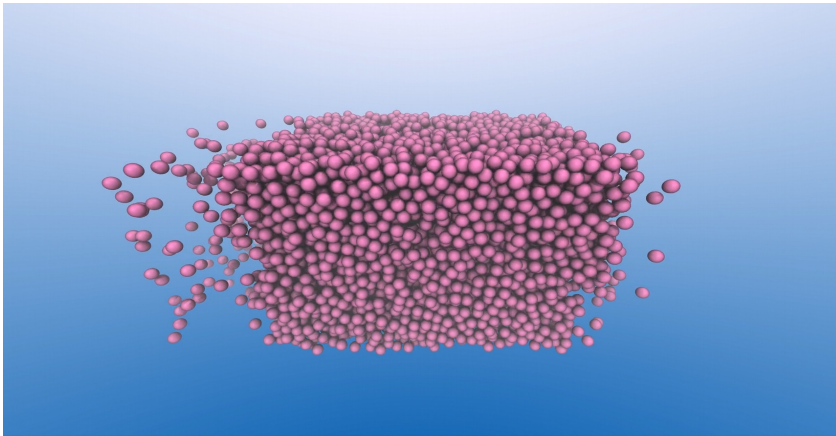




# Long-range correction at planar interfaces

For planar interfaces:

**Long-range correction** from the density profile, following **Janeček**.

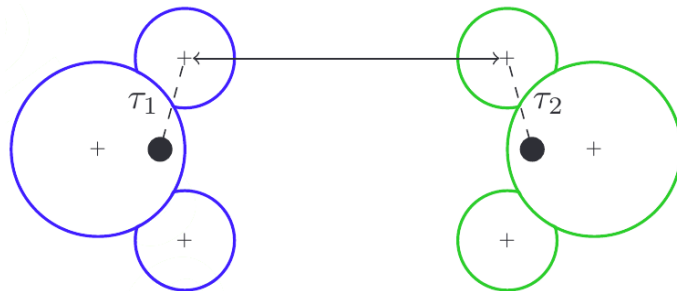


Full evaluation of all pairwise interactions is too expensive ...  
 ... instead, **short-range interactions** are evaluated for **neighbors**.

# Long-range correction at planar interfaces

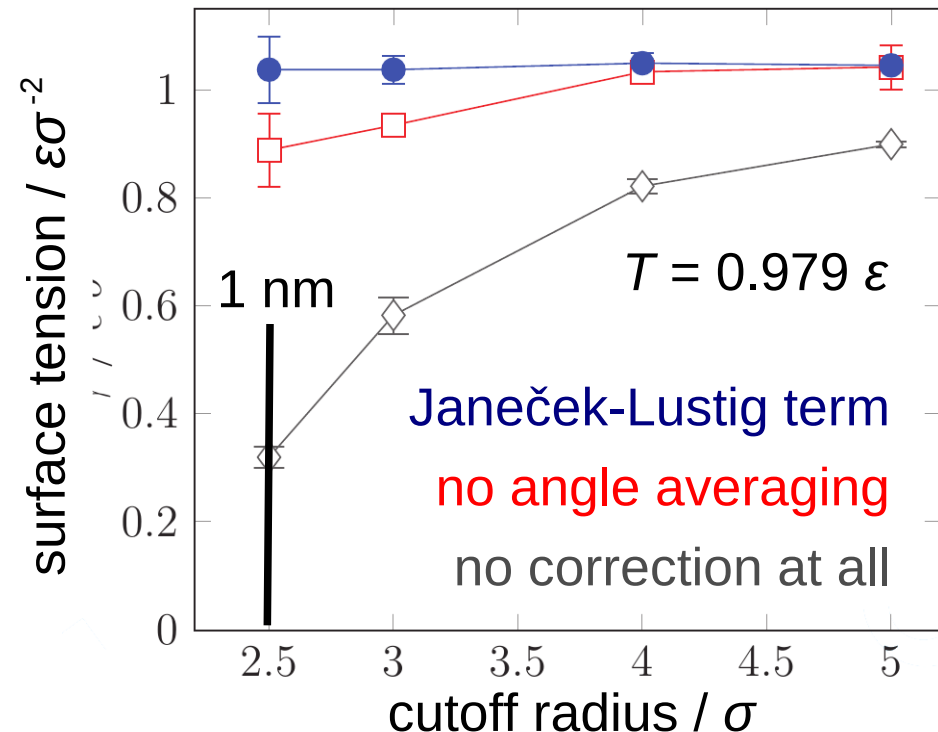
For planar interfaces:

Long-range correction from the density profile, following **Janeček**.



Angle-averaging expression for multi-site models, following **Lustig**.

Two-center LJ fluid (2CLJ)

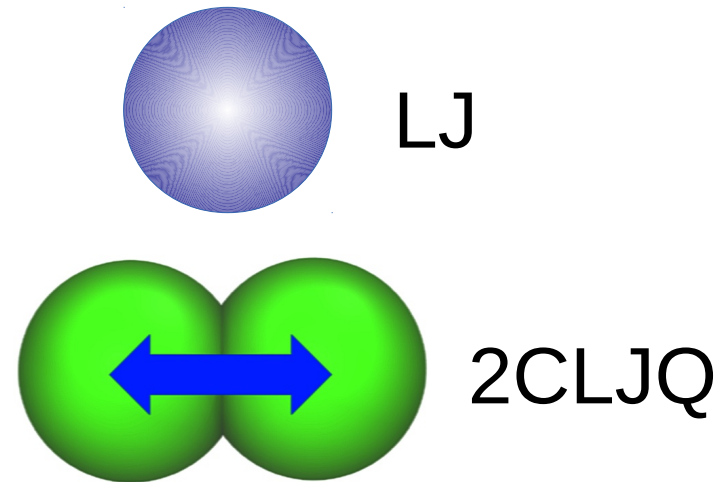
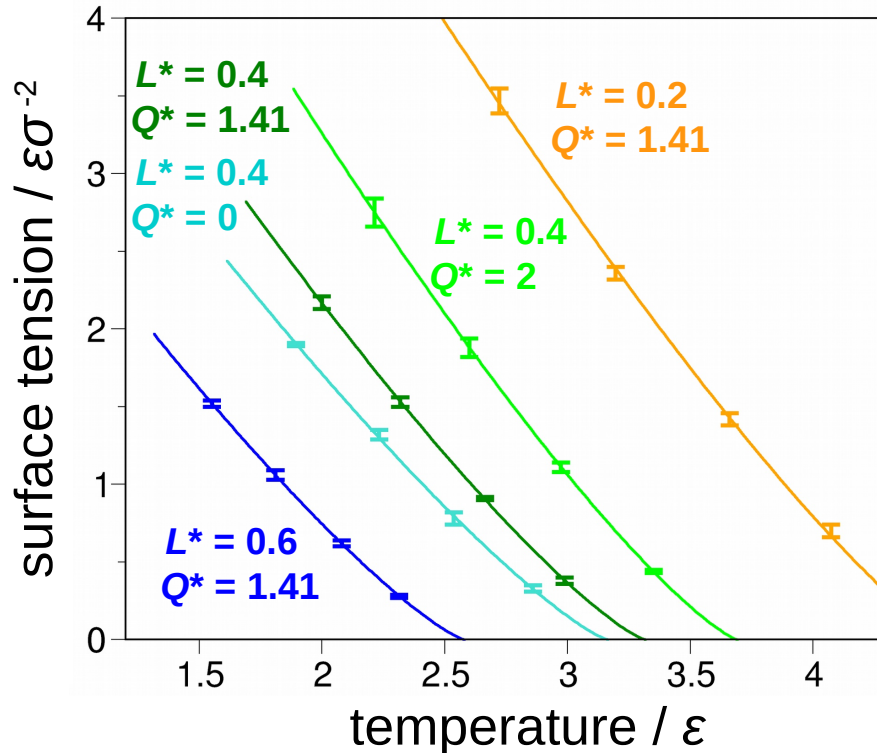


For arbitrary geometries, e.g. the fast multipole method can be employed.



# Massively parallel molecular modeling

Two LJ + quadrupole (2CLJQ)



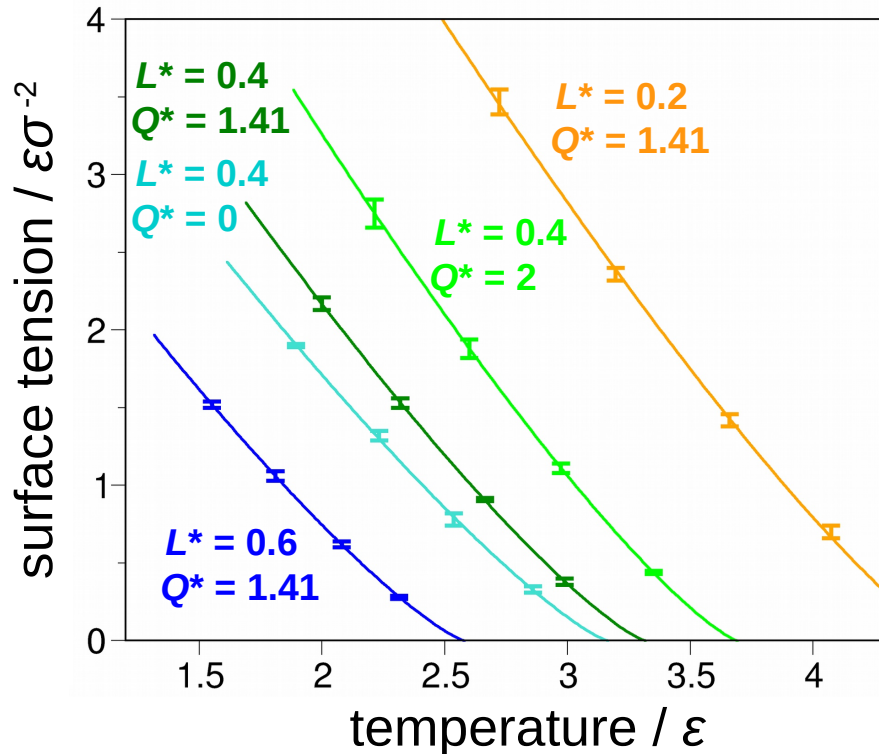
- Systematic exploration of the four-dimensional model parameter space



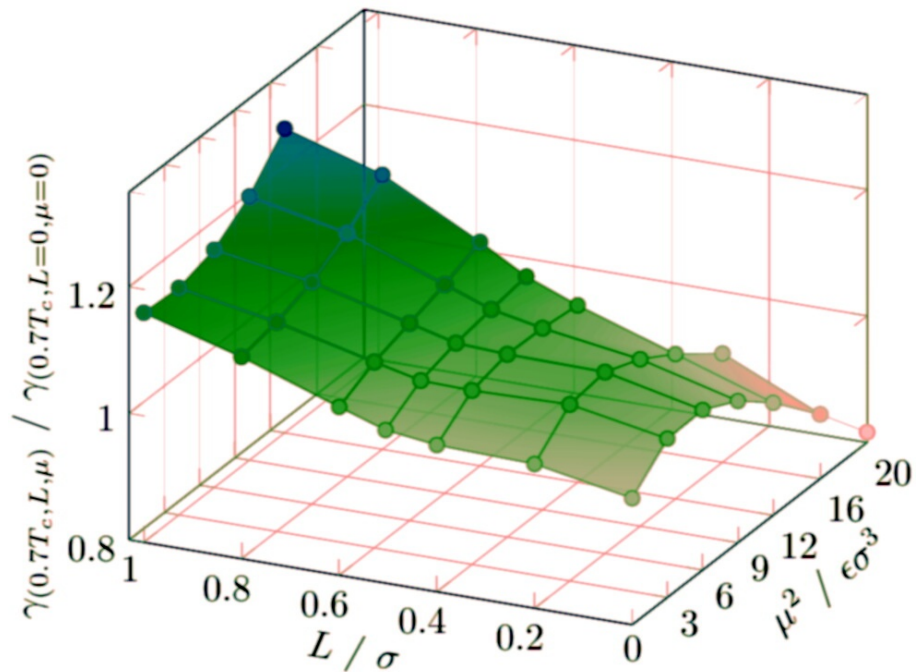


# Massively parallel molecular modeling

Two LJ + quadrupole (2CLJQ)



Two LJ + dipole (2CLJD)



- Systematic exploration of the four-dimensional model parameter space
- Correlation of the surface tension by a critical scaling expression

# Parameterization of molecular force fields

## Geometry

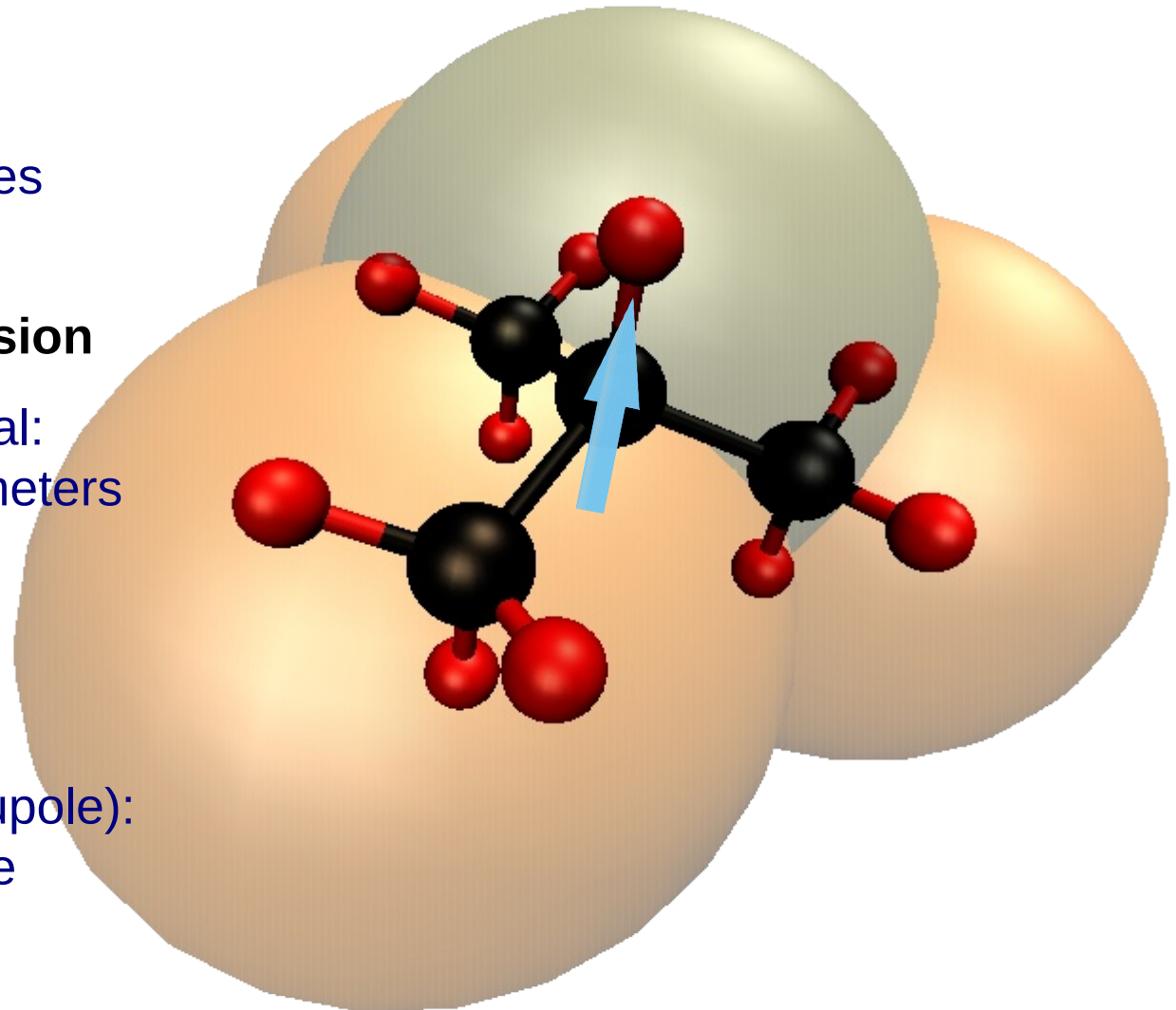
Bond lengths and angles

## Dispersion and repulsion

Lennard-Jones potential:  
Size and energy parameters

## Electrostatics

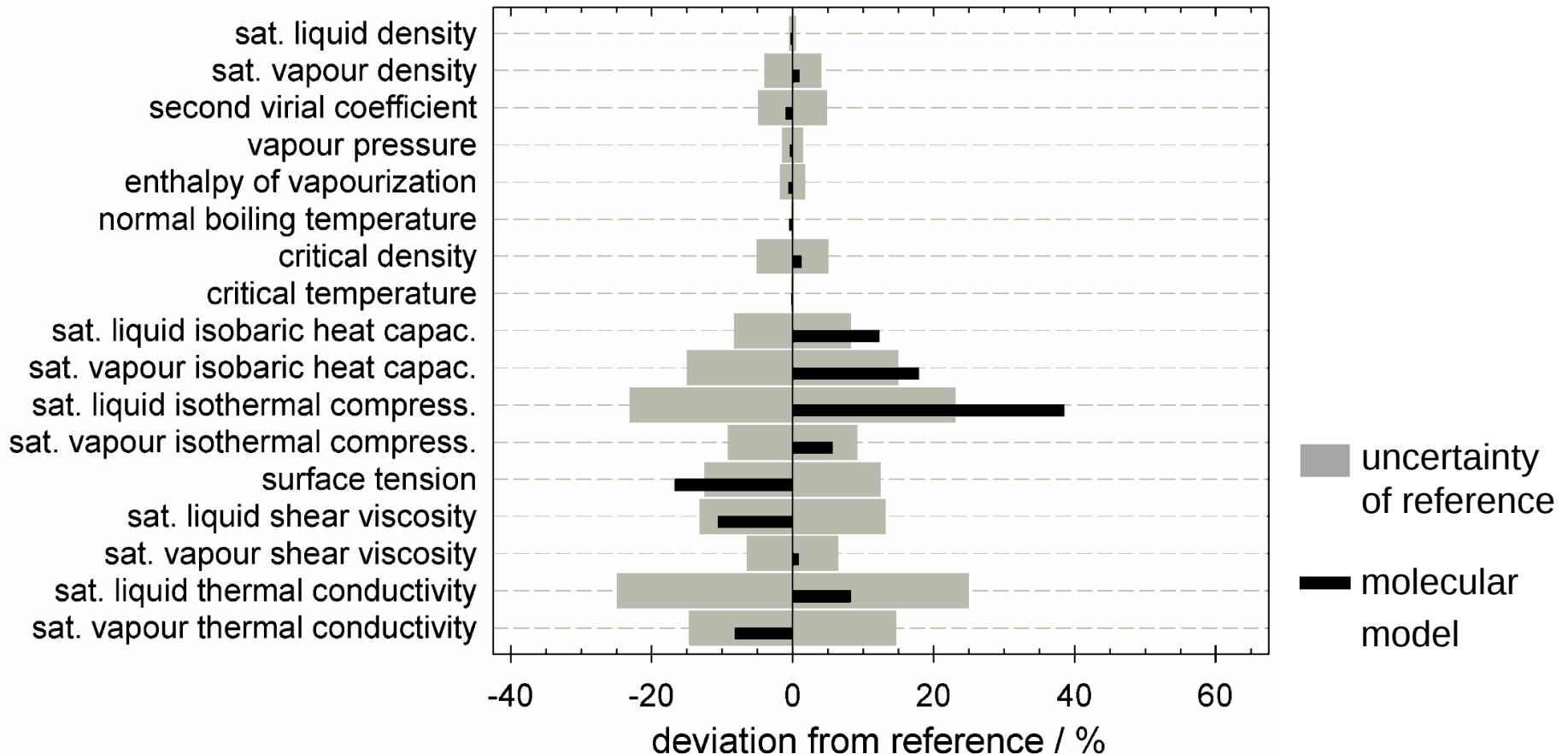
Point polarities  
(charge, dipole, quadrupole):  
Position and magnitude





# Quantitative reliability of molecular models

ethylene oxide model by Eckl *et al.* (2008)

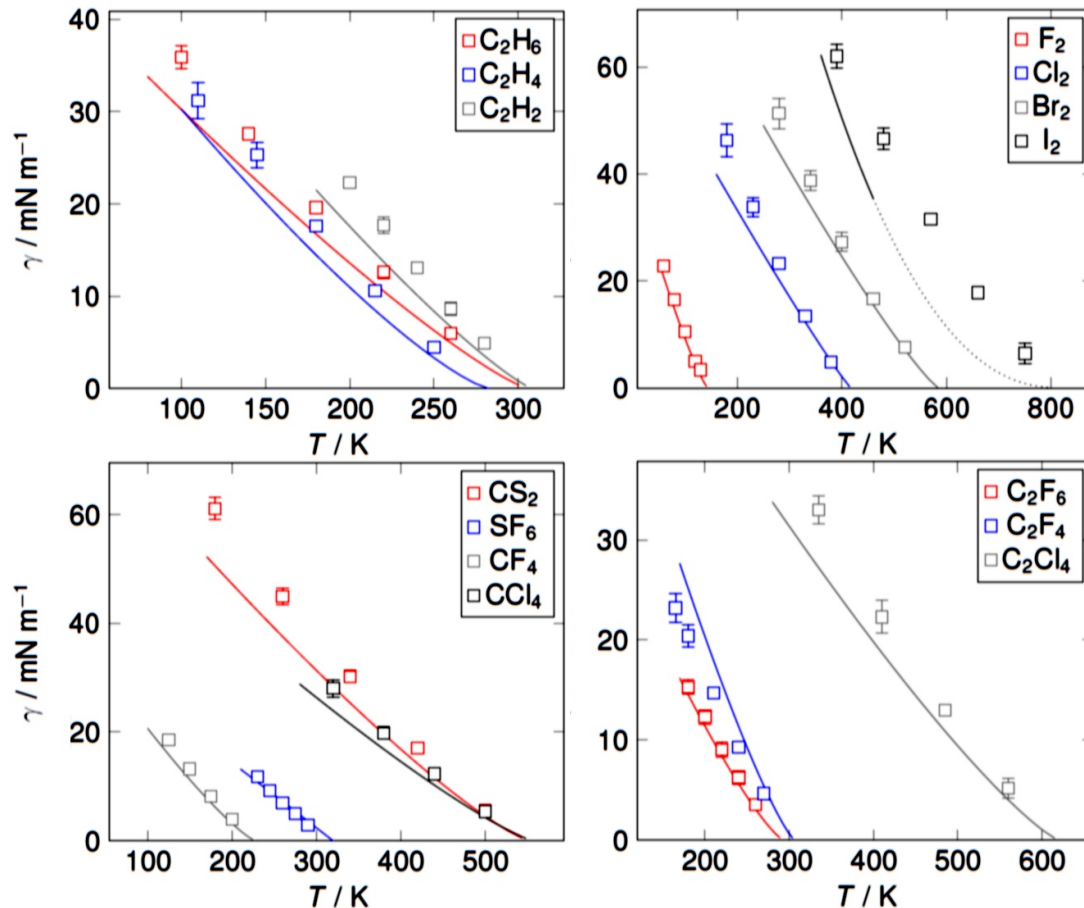




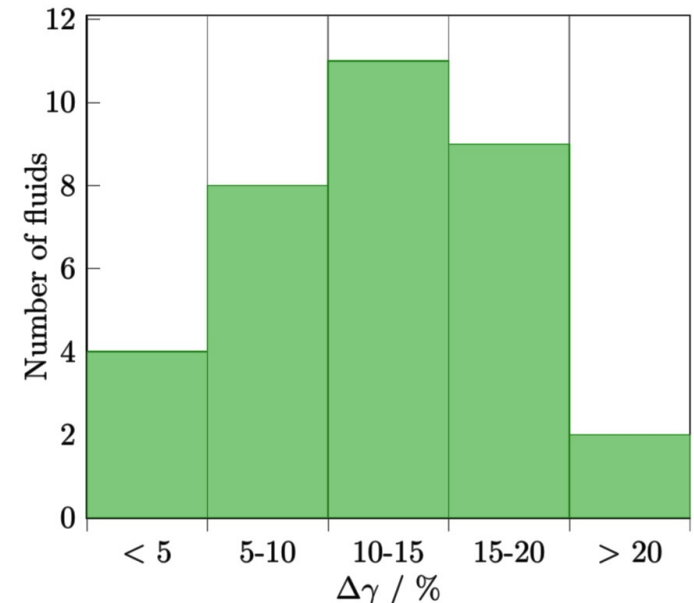



# Predictive capacity of literature models

Two LJ + quadrupole (2CLJQ)



Two LJ + dipole (2CLJD)

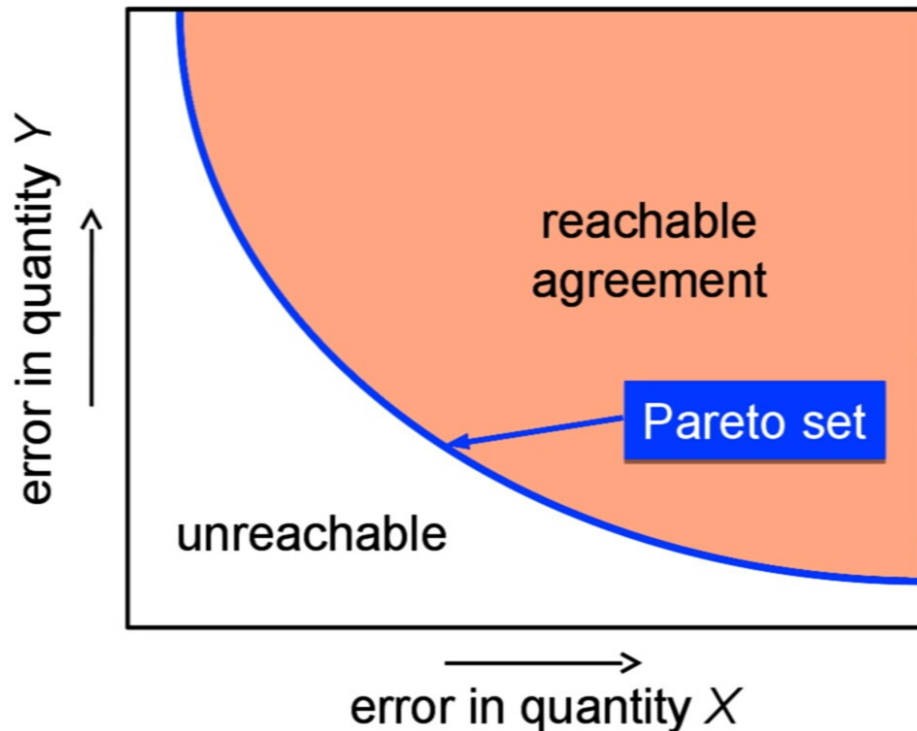


Fit to bulk properties   
10 to 20 % overestimation of  
vapor-liquid surface tension

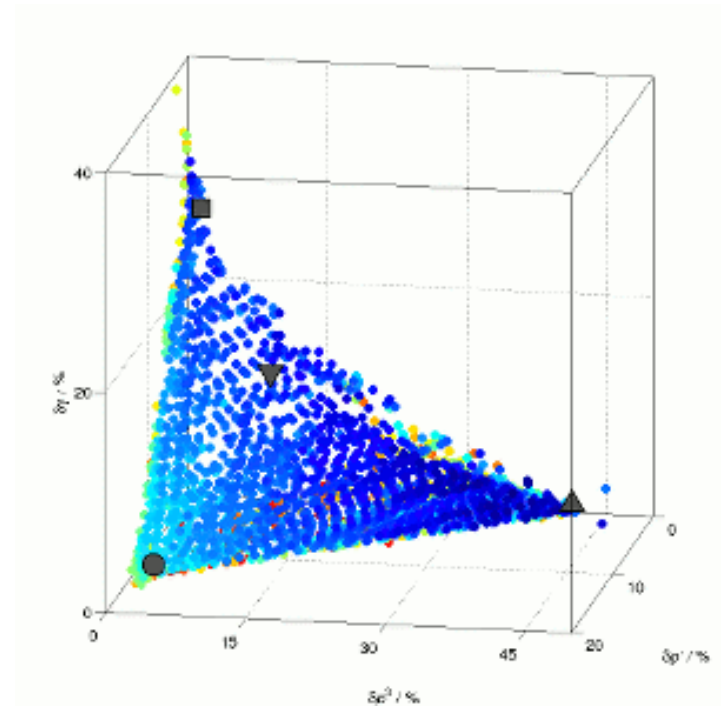


# Model tailoring by multicriteria optimization

Pareto optimality criterion



Pareto set for carbon dioxide

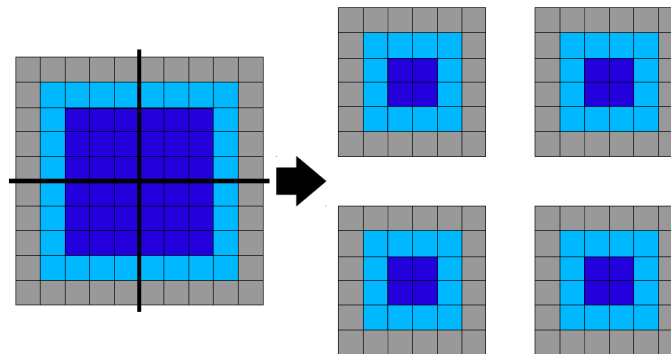


Multicriteria optimization requires massively-parallel molecular modeling.



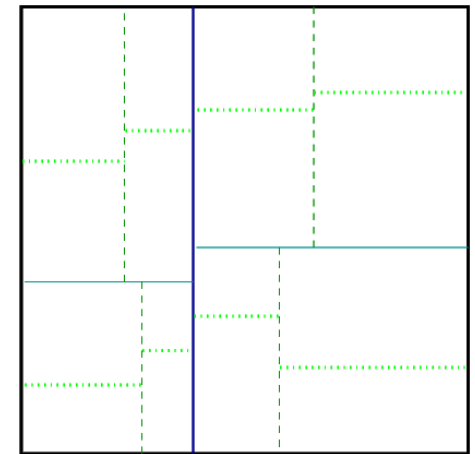
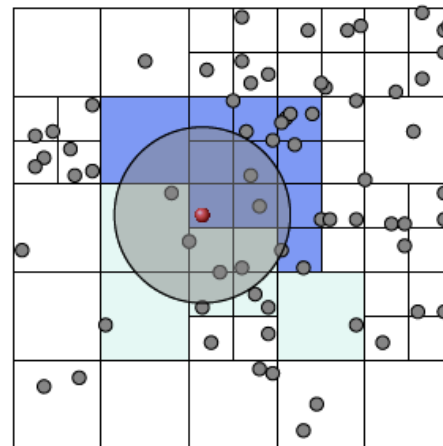
# Efficient simulation of large systems

Linked-cell data structure  
suitable for spatial domain  
decomposition:



(non-blocking, over-  
lapping MPI send/  
receive operations)

Methods for heterogeneous  
or fluctuating particle  
distributions:



large systems “1”: molecular dynamics

<http://www.ls1-mardyn.de/>



# Efficient simulation of large systems

Memory-efficient implementation based on the linked-cell data structure:

21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

sliding window

Optionally, forces acting on molecules are only stored until their cell leaves the sliding window.

large systems “1”: molecular dynamics

<http://www.ls1-mardyn.de/>



# Efficient simulation of large systems

Memory-efficient implementation based on the linked-cell data structure:

21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

hyperthreaded sliding window

Optionally, forces acting on molecules are only stored until their cell leaves the sliding window.

Efficient vectorization:

- Optimization by hand, using advanced vector extensions (AVX).
- Conversion from array of structures (AoS) to structure of arrays (SoA).

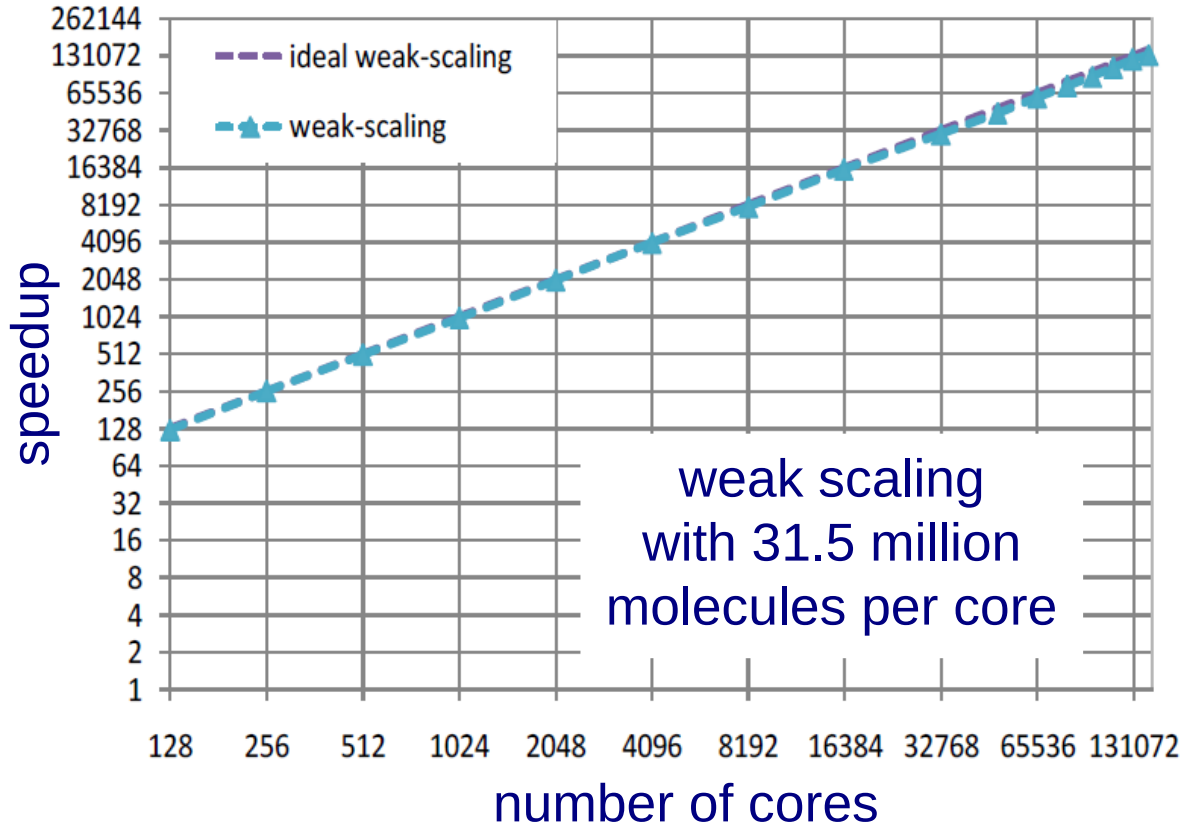
large systems “1”: molecular dynamics

<http://www.ls1-mardyn.de/>





# Large-scale MD simulation on SuperMUC



Up to  $N = 4 \cdot 10^{12}$   
on SuperMUC



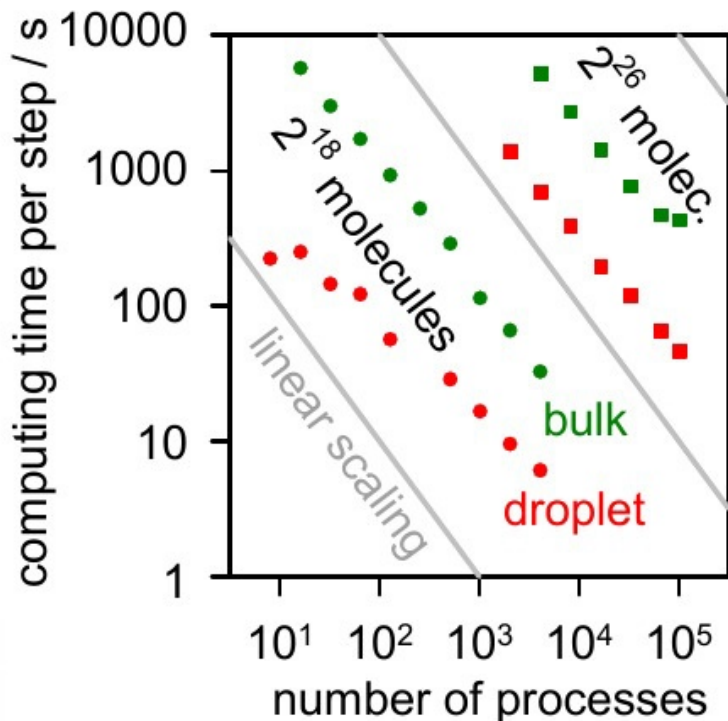
large systems “1”: molecular dynamics

<http://www.ls1-mardyn.de/>

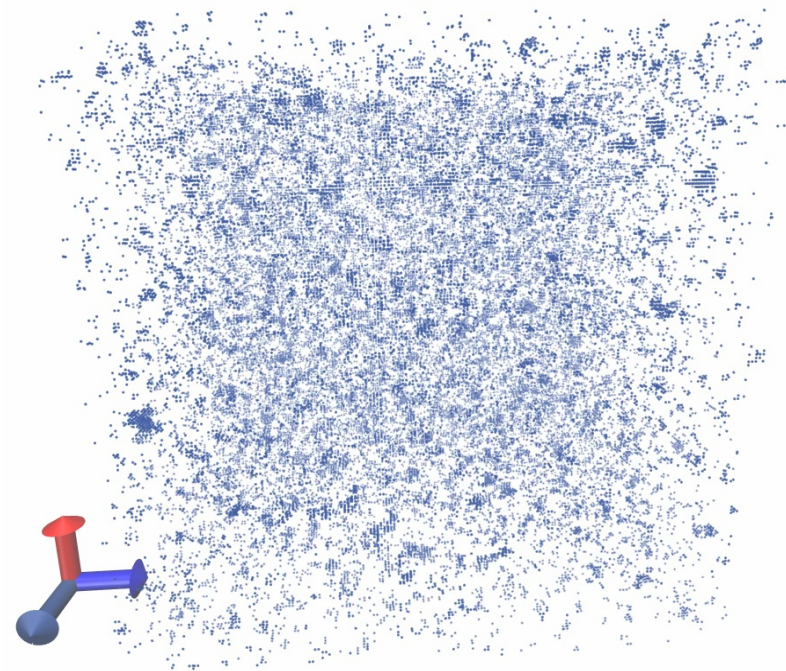


# Large-scale MD simulation on hermit

strong scaling (Amdahl)



homogeneous cavitation

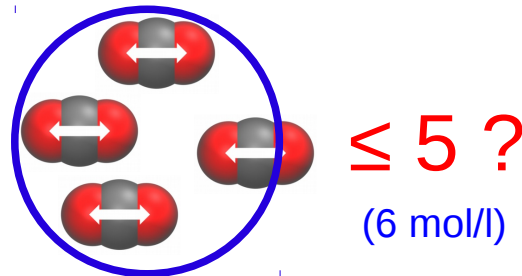


CO<sub>2</sub> ( $T = 280$  K and  $\rho = 17.2$  mol/l), 3CLJQ  
 25 million molecules on 110 592 cores

# Large-scale MD simulation of cavitation

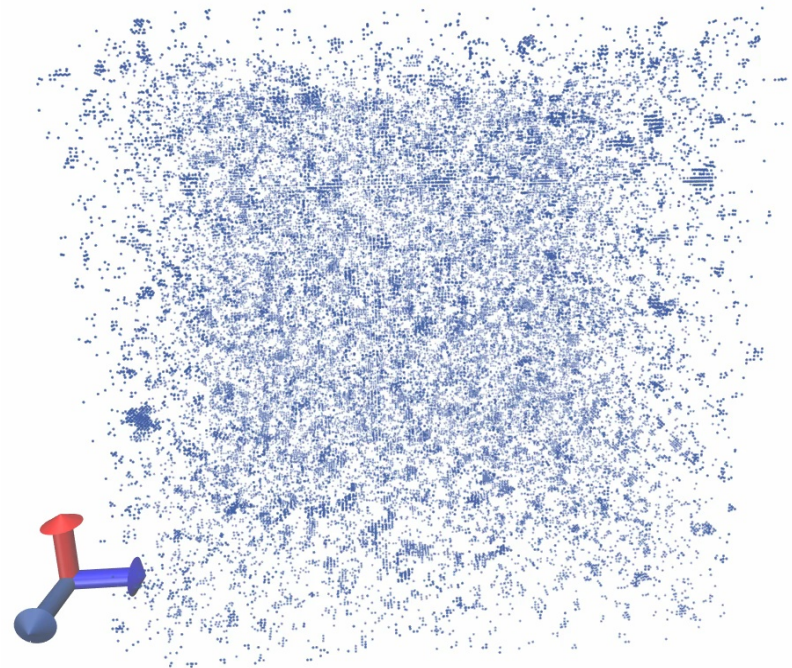
Scale-up to the entire *hermit* cluster for canonical simulation of cavitation in carbon dioxide.

Evaluation of local density at 180 x 180 x 180 grid points:



Liquid phase detected for more than 5 neighbors within a radius of 6.9 Å around the grid point.

homogeneous cavitation



CO<sub>2</sub> ( $T = 280$  K and  $\rho = 17.2$  mol/l), 3CLJQ  
25 million molecules on 110 592 cores

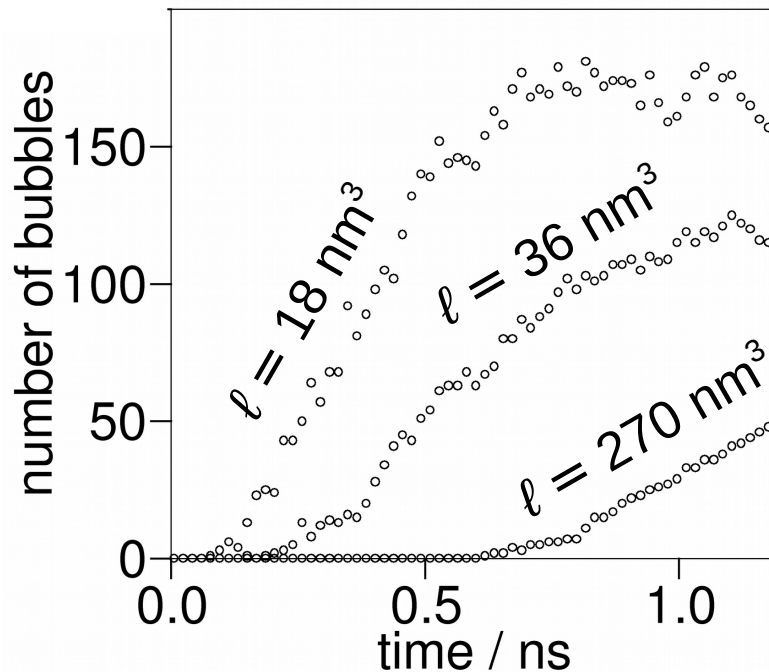




# Cavitation in a metastable liquid phase

Yasuoka-Matsumoto method: Count nuclei exceeding a threshold size  $\ell$ .

$$N = 2.5 \times 10^7, V = 2.41 \times 10^{-21} \text{ m}^3, T = 280 \text{ K}$$



Three consecutive regimes:

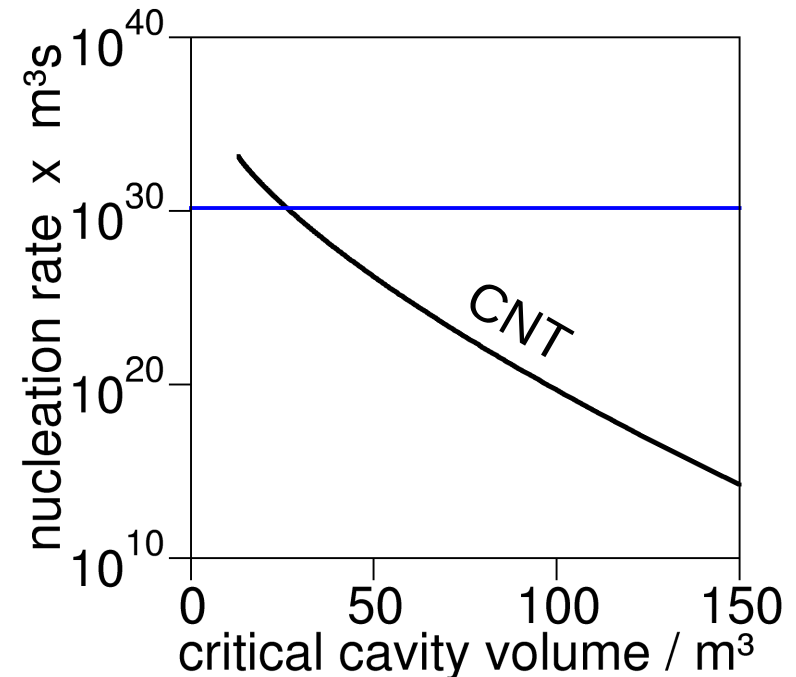
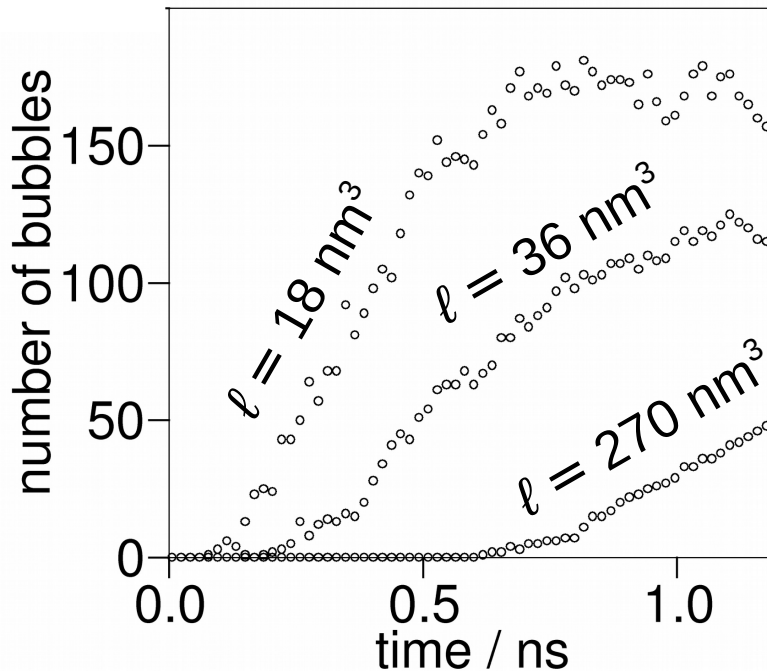
- relaxation (equilibration)
- homogeneous cavitation
- growth and aggregation



# Cavitation in a metastable liquid phase

Yasuoka-Matsumoto method: Count nuclei exceeding a threshold size  $\ell$ .

$$N = 2.5 \times 10^7, V = 2.41 \times 10^{-21} \text{ m}^3, T = 280 \text{ K}$$

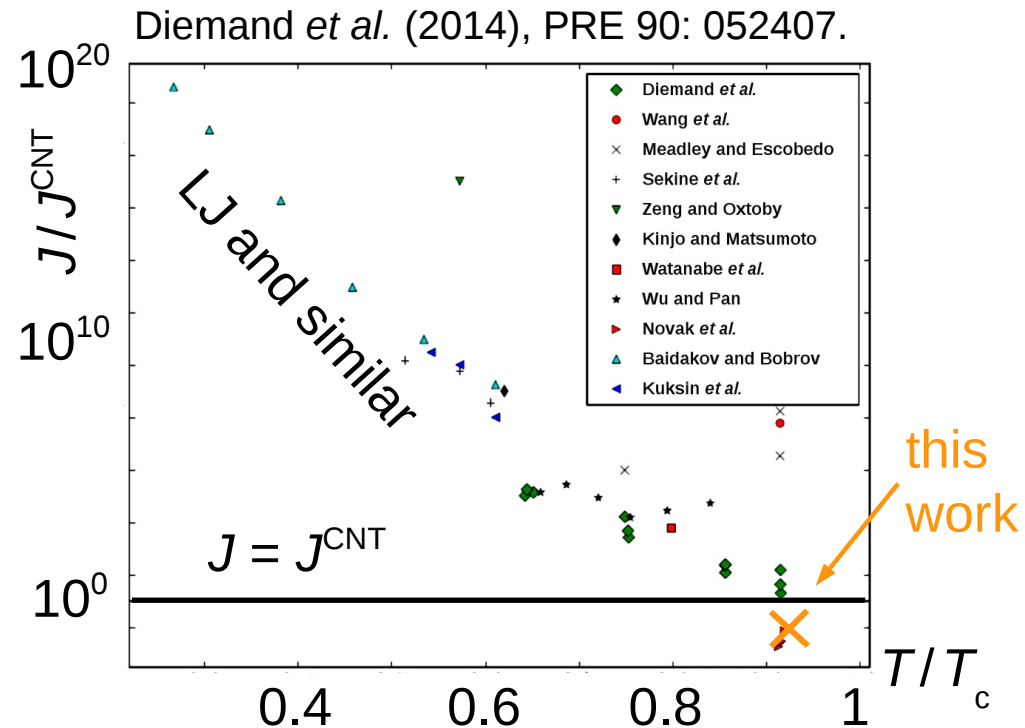
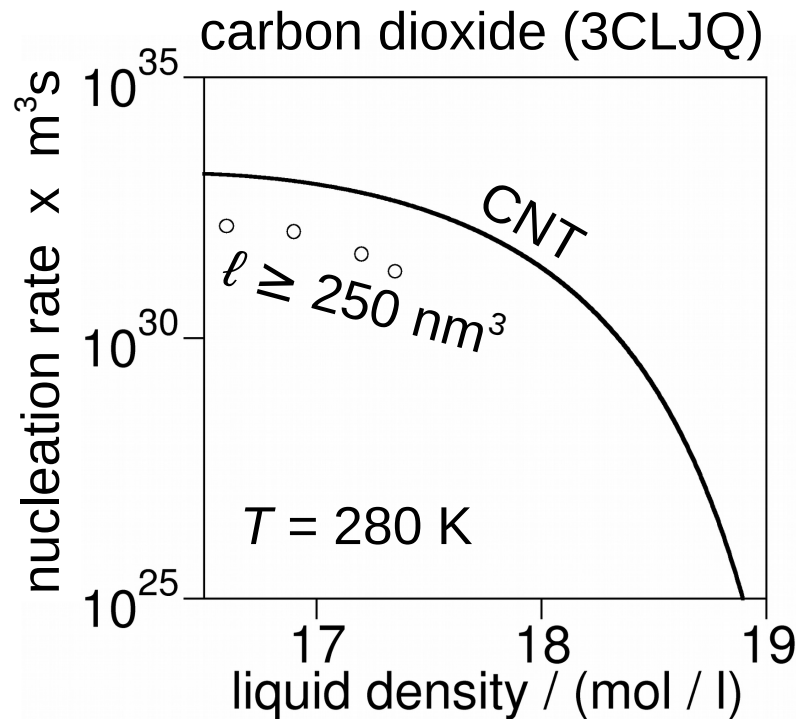


Classical nucleation theory predicts critical cavity sizes from 10 to 30 nm<sup>3</sup>.



# MD simulation ./ classical nucleation theory

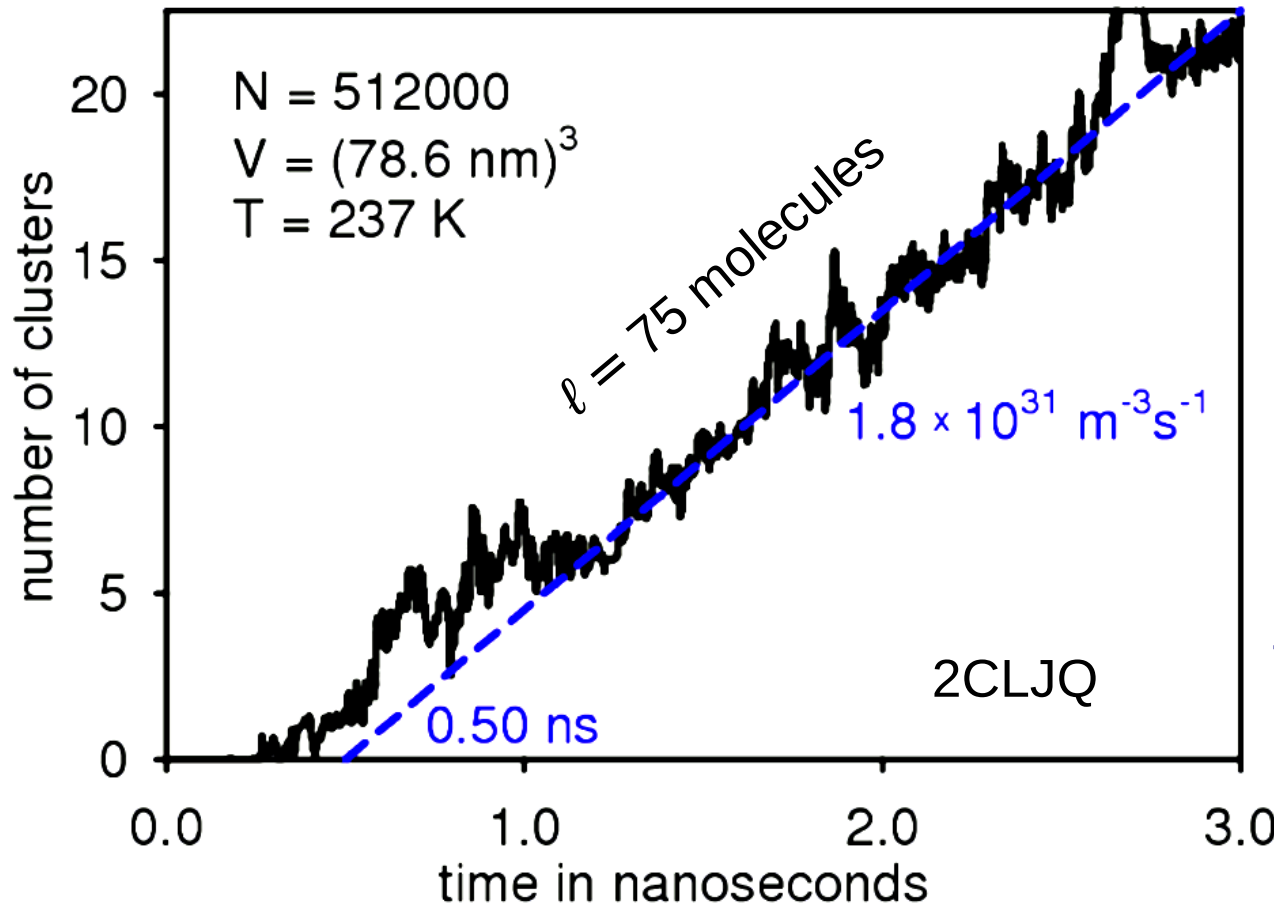
Cavities with a volume greater than  $250 \text{ nm}^3$  are sufficiently supercritical.



For cavitation at high temperatures, CNT is a good approximation.



# Nucleation in supersaturated vapors



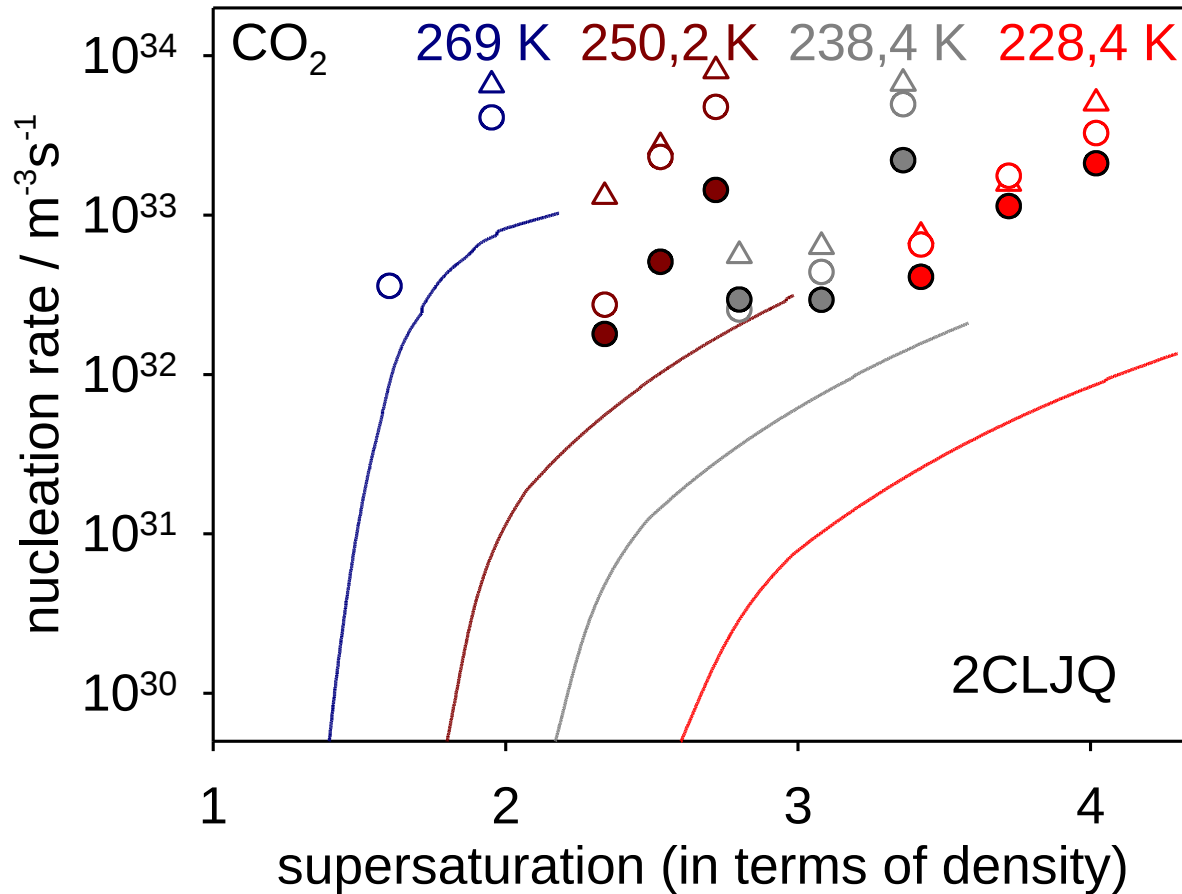
Cluster criterion:

Stillinger type, single  
 neighbor within radius  
 $1.5 \sigma + L/4$ , i.e.  $5.1 \text{ \AA}$ .

Critical size predicted  
 by CNT in region of  
 interest:

40 to 60 molecules.

# Nucleation in supersaturated vapors



Cluster criterion:

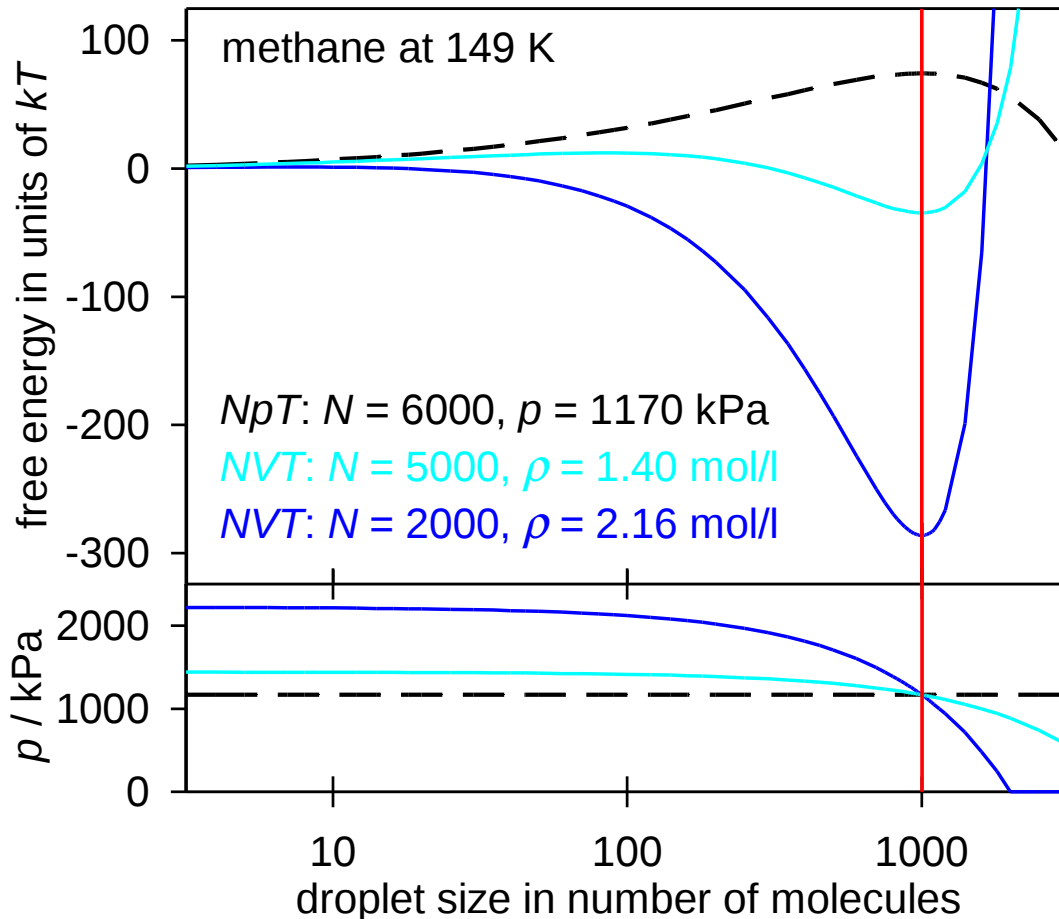
Stillinger type, single  
 neighbour within radius  
 $1.5 \sigma + L/4$ , i.e. 5.1 Å.

— CNT (nonisothermal)

- $\triangle$   $l = 50$  molecules
- $\circ$   $l = 75$  molecules
- $\bullet$   $l = 250$  molecules



# Stable and unstable phase equilibria



Equilibrium condition for a nucleus of size  $j$ :

$$p = p(T, j)$$

$\Delta G$  at constant  $p$  and  $T$ :

1 unstable equilibrium

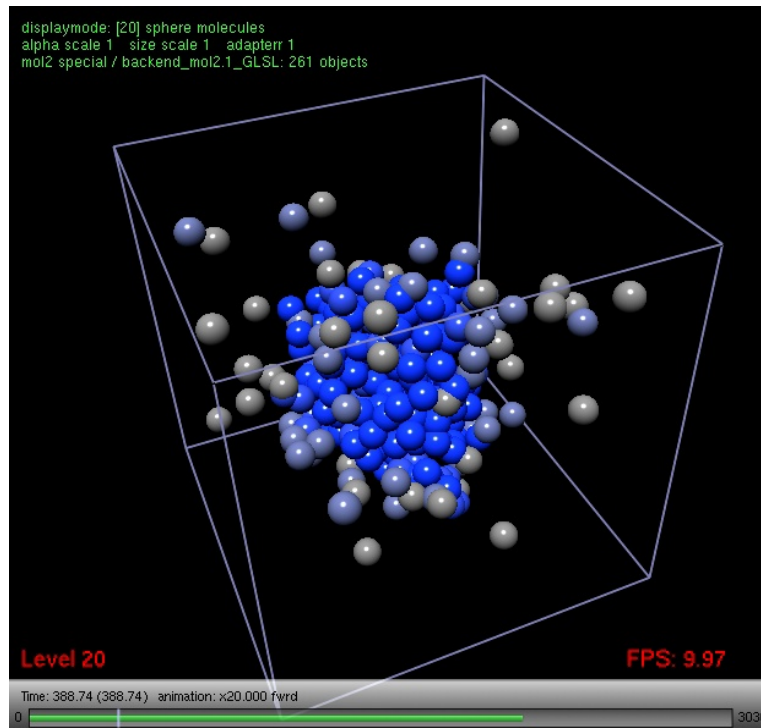
$\Delta F$  at constant  $V$  and  $T$ :

(1 unstable equilibrium)

1 stable equilibrium



# Molecular simulation of nanodroplets



Equilibrium condition for  
a nucleus of size  $j$ :

$$p = p(T, j)$$

$\Delta G$  at constant  $p$  and  $T$ :

1 unstable equilibrium

$\Delta F$  at constant  $V$  and  $T$ :

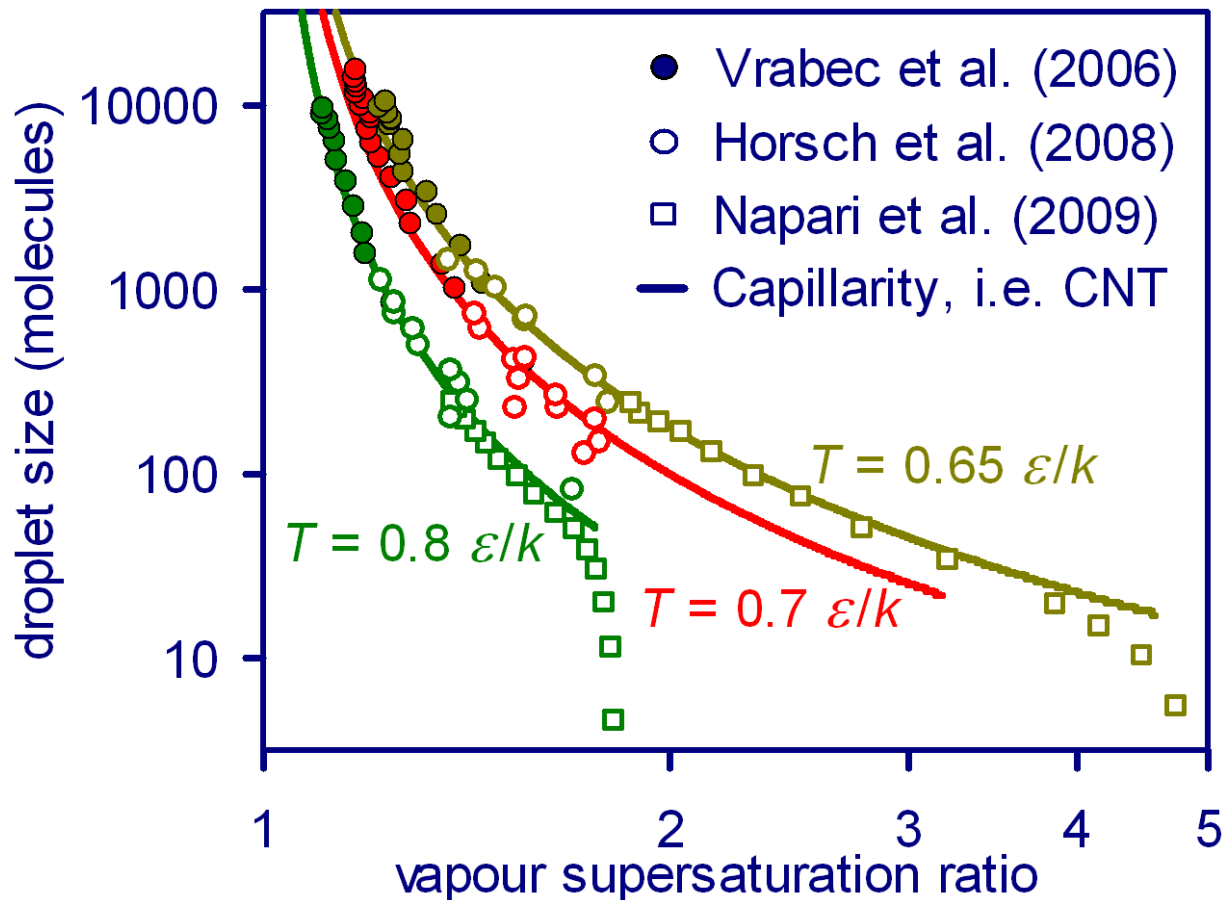
(1 unstable equilibrium)

1 stable equilibrium



# Molecular simulation of nanodroplets

Canonical MD simulation of LJTS droplets



Down to 100 molecules: agreement with CNT ( $\gamma = \gamma_0$ ).

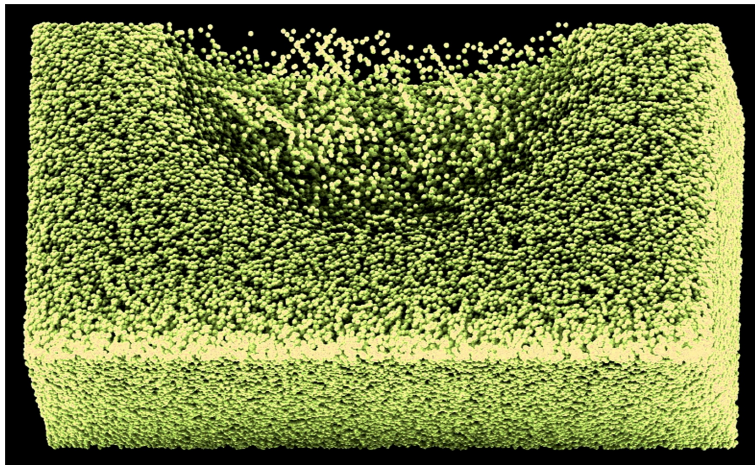
At the spinodal, the results suggest that  $R_\gamma = 2\gamma / \Delta p \rightarrow 0$ . This implies

$$\lim_{R_\gamma \rightarrow 0} \gamma = 0,$$

as conjectured by Tolman (1949) ...

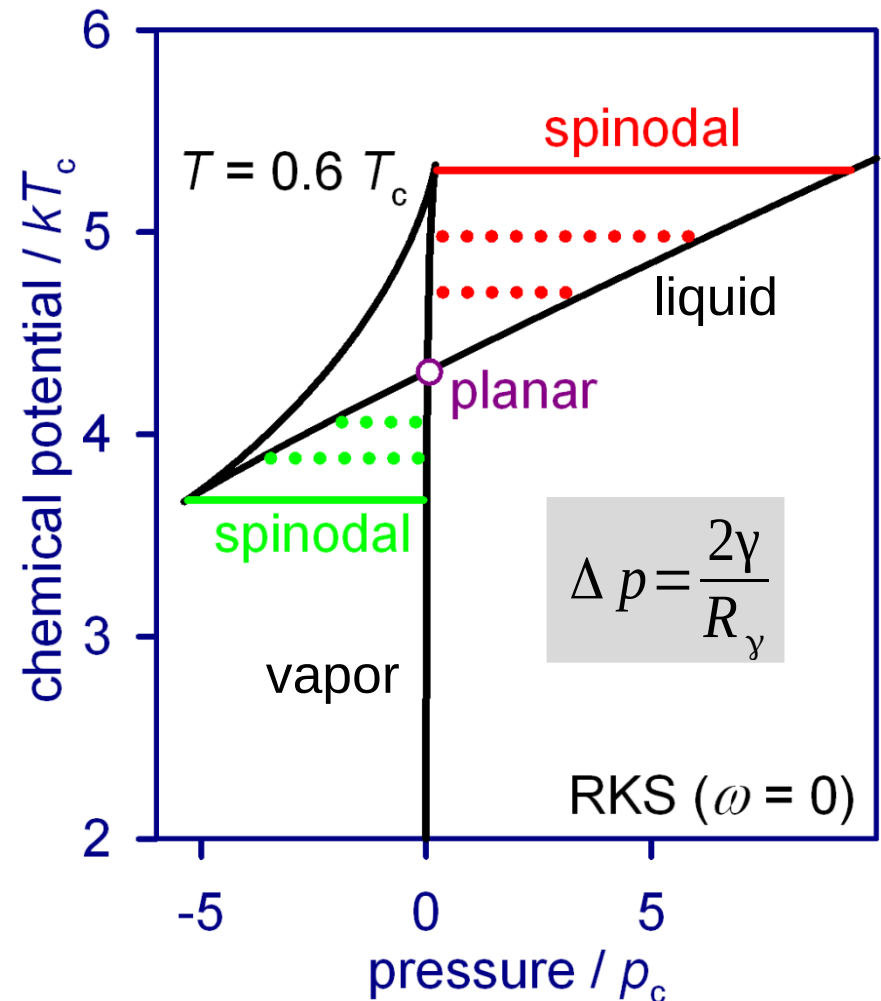
# Molecular simulation of nanobubbles

- Droplet + metastable vapor
- Bubble + metastable liquid



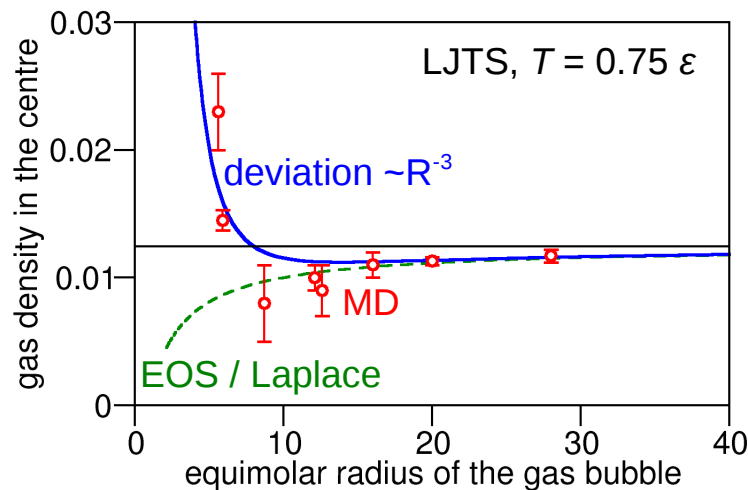
**Spinodal limit:** For the external phase, metastability breaks down.

**Planar limit:** The curvature changes its sign and the radius  $R_y$  diverges.



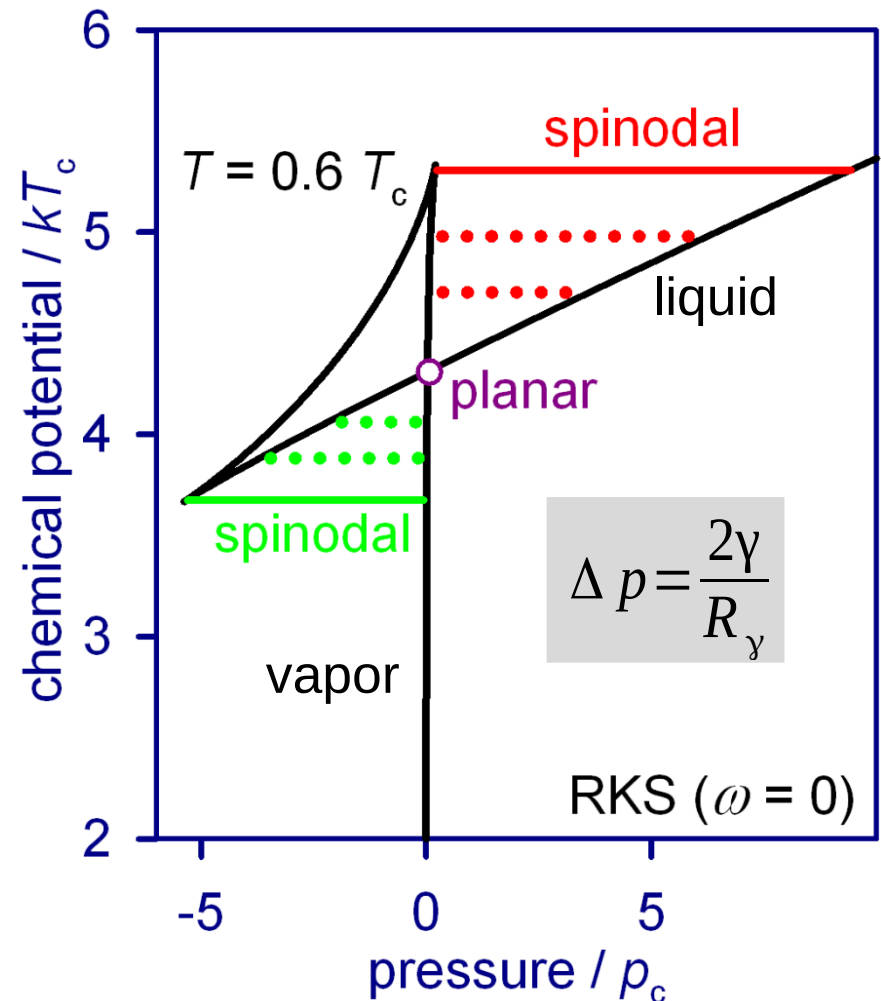
# Molecular simulation of nanobubbles

- Droplet + metastable vapor
- Bubble + metastable liquid



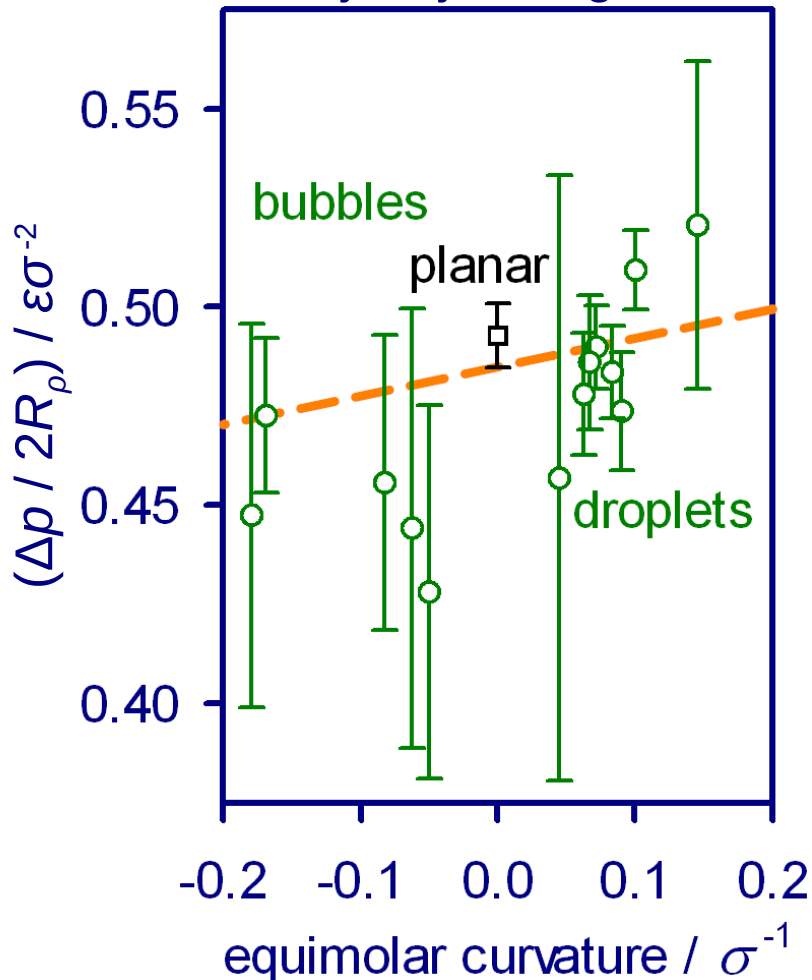
**Spinodal limit:** For the external phase, metastability breaks down.

**Planar limit:** The curvature changes its sign and the radius  $R_y$  diverges.



# Interpolation to the planar limit

Nijmeijer diagram



- Convention: Negative curvature (bubbles), positive curvature (droplets).
- Properties of the planar interface, such as its Tolman length, can be obtained by interpolation to zero curvature.
- A positive slope of  $\Delta p / 2R_\rho$  over  $1/R_\rho$  in the Nijmeijer diagram corresponds to a **negative**  $\delta$ , on the order of  $-0.1 \sigma$  here, conforming that  $\delta$  is small.
- However,  $\mathbf{R} \rightarrow \mathbf{0}$  for droplets in the spinodal limit for the surrounding vapor (Napari *et al.*) implies  $\mathbf{y} \rightarrow \mathbf{0}$ .



# Breakdown of Tolman's law

The relation obtained by Tolman (1949) for the surface tension of droplets

$$\frac{d \ln R}{d \ln \gamma} = 1 + \frac{1}{2} \left( \frac{\delta}{R} + \left[ \frac{\delta}{R} \right]^2 + \frac{1}{3} \left[ \frac{\delta}{R} \right]^3 \right) \quad \longrightarrow \quad \frac{\gamma}{\gamma_0} = 1 - \frac{2\delta}{R} + O \left( \left[ \frac{\delta}{R} \right]^2 \right)$$

- accounts for an influence of the radius on the surface tension, but not for *curvature-independent effects* (by construction),
- assumes that the dependence of the surface tension on curvature is *monotonous* (except if the Tolman length  $\delta$  changes its sign),
- is derived from a thermodynamic approach based on the *Gibbs adsorption equation*, i.e.  $d\gamma = -\Gamma d\mu$ , at constant temperature.





# The adsorption equation for macrosystems

A comparison of the total *differential* for the free energy

$$dF = \gamma dA - P' dV' - P'' dV'' - S dT + \sum \mu_i dN_i$$

with the *absolute* free energy  $F = \gamma A - P' V' - P'' V'' + \sum \mu_i N_i$  yields:

$$A d\gamma = V' dP' + V'' dP'' - S dT - \sum N_i d\mu_i$$

$$0 = V' dP' - S' dT - \sum N_i' d\mu_i$$

$$0 = V'' dP'' - S'' dT - \sum N_i'' d\mu_i$$

$$A d\gamma = -\eta dT - A \sum \Gamma_i d\mu_i$$

It follows that, at constant temperature,

$$d\gamma = - \sum \Gamma_i d\mu_i. \quad (\text{Gibbs adsorption equation})$$



# The adsorption equation for nanosystems

A comparison of the total *differential* for the free energy

$$dF = \gamma dA - P' dV' - P'' dV'' - S dT + \sum \mu_i dN_i$$

with the *absolute* free energy  $F = \sigma A - P' V' - P'' V'' + \sum \mu_i N_i$  yields:

$$A d\sigma + (\sigma - \gamma) dA = V' dP' + V'' dP'' - S dT - \sum N_i d\mu_i$$

$$0 = V' dP' - S' dT - \sum N_i' d\mu_i$$

$$0 = V'' dP'' - S'' dT - \sum N_i'' d\mu_i$$

$$A d\sigma + (\sigma - \gamma) dA = -\eta dT - A \sum \Gamma_i d\mu_i$$

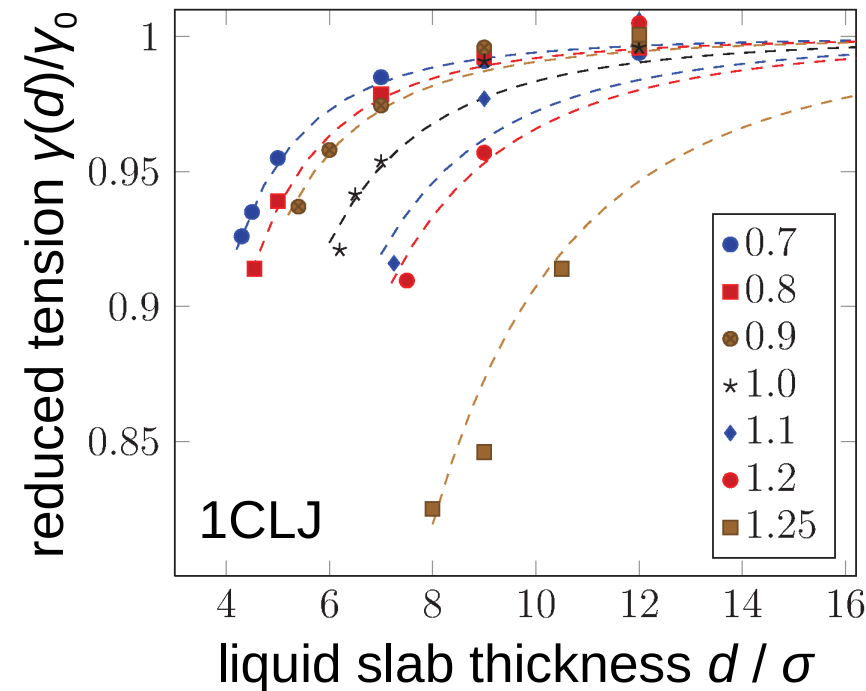
It follows that ...

finite-size effects with  $\sigma \neq \gamma$  invalidate the Gibbs adsorption equation.



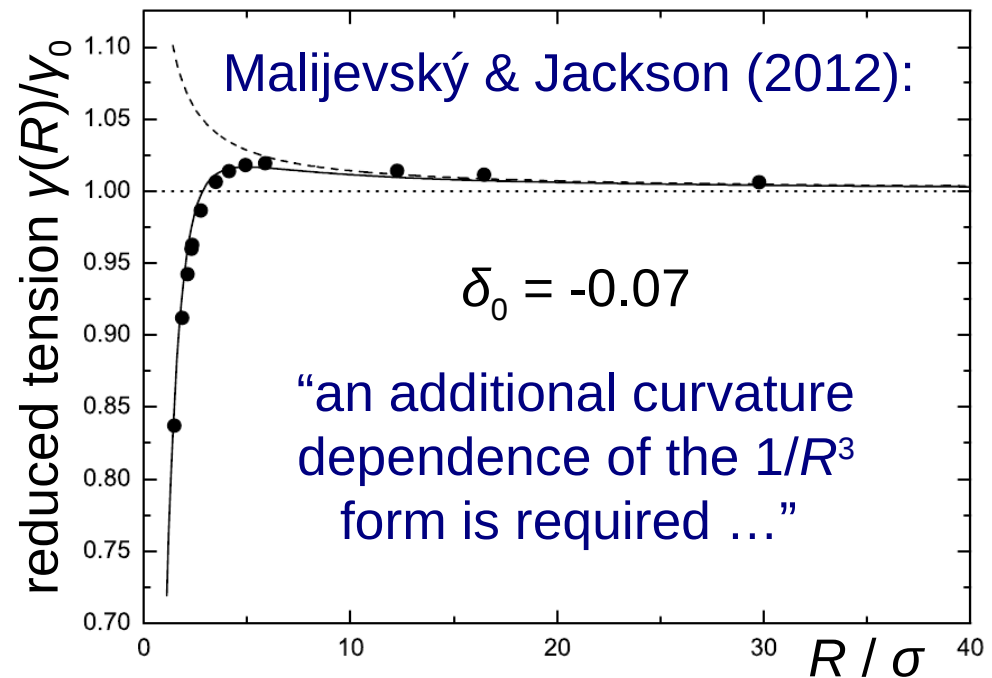
# Molecular simulation of thin liquid films

Surface tension of thin films:



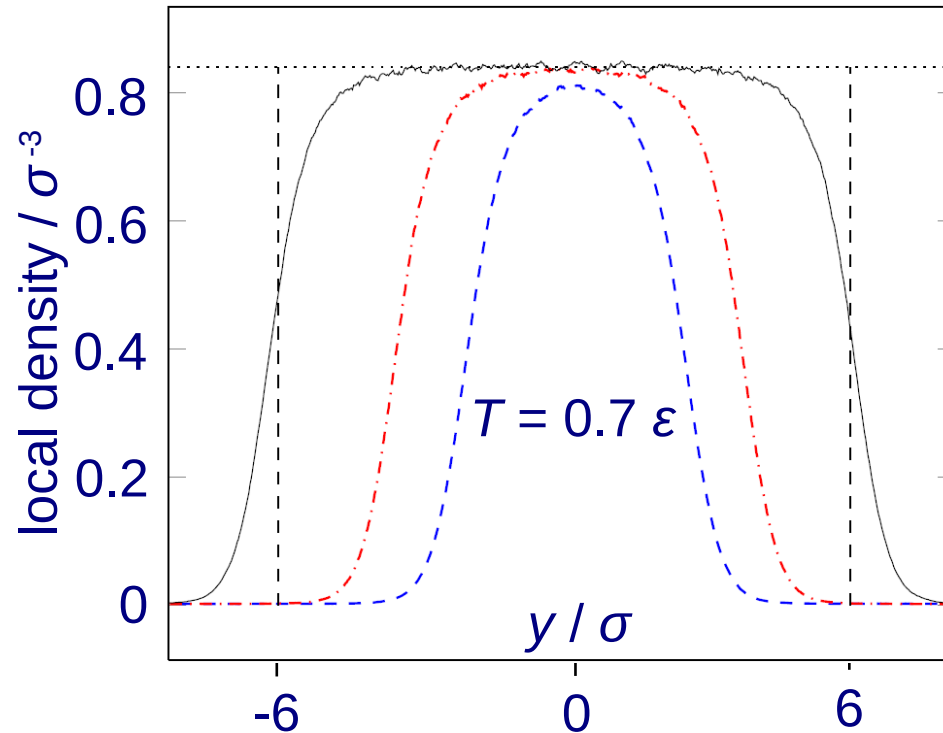
Relation with  $\gamma(R)$  for droplets?

$\delta_0$  is small and probably negative



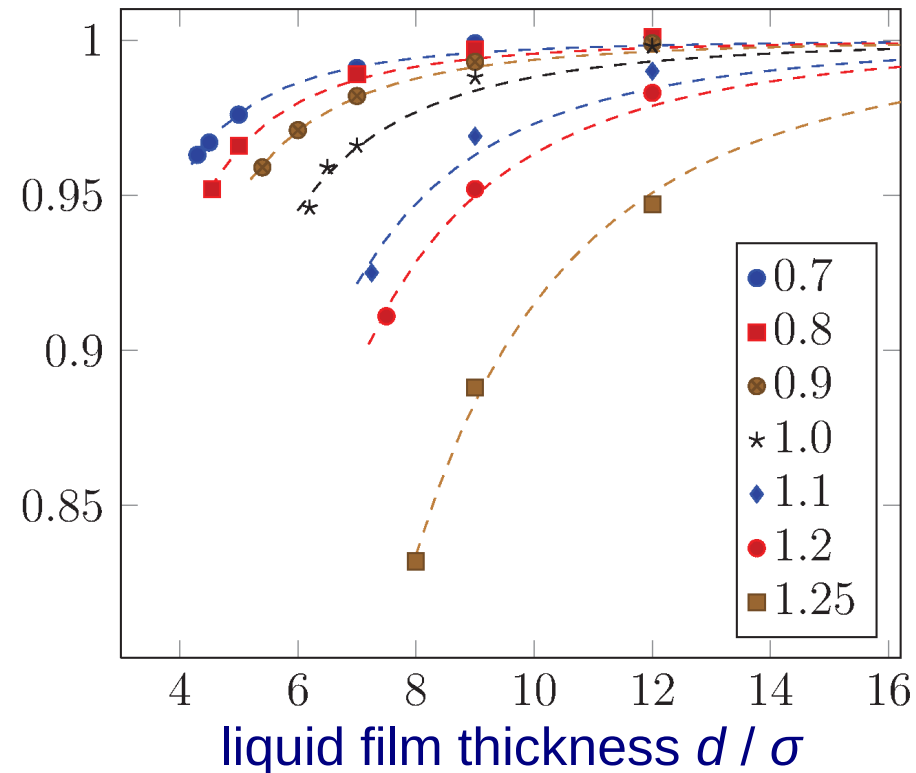


# Molecular simulation of thin liquid films

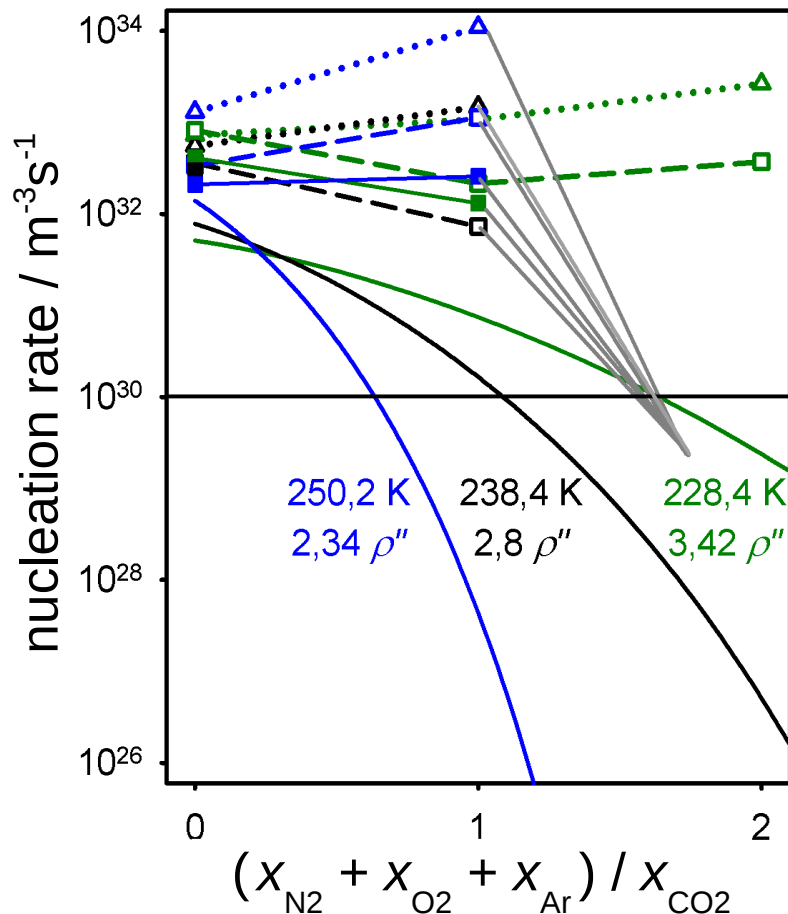


The density in the center of liquid nanofilms deviates from that of the saturated bulk liquid.

By simulating thin liquid films, curvature-independent size effects can be considered.



# Nucleation in mixtures: Air + carbon dioxide



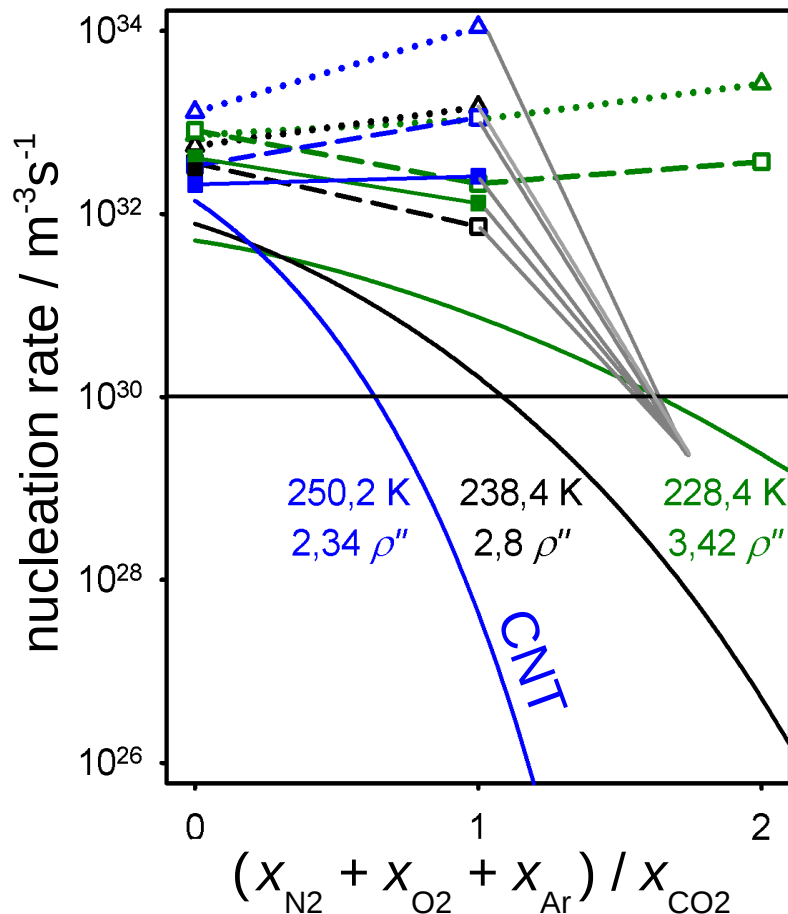
Scenario:

- Vapor contains four components
- Bulk liquid mainly contains  $\text{CO}_2$

Results from MD simulation:

$\triangle \ell = 50$      $\square \ell = 100$      $\blacksquare \ell = 150$

# The carrier gas effect on nucleation



Scenario:

- Vapor contains  $k$  components
- Liquid phase is approximately pure
- Other  $k - 1$  components: Carrier gas

Carrier gas effect (Wedekind *et al.*):

- Thermalization  $\rightarrow J$  increases
- Greater pressure  $\rightarrow J$  decreases

— CNT following Wedekind *et al.*

$\triangle$   $l = 50$

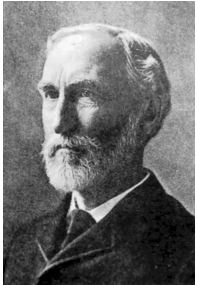
$\square$   $l = 100$

$\blacksquare$   $l = 150$





# Interfacial enrichment of carrier gases



„take some point [...] and **imagine a geometrical surface** to pass through this point and all other points which are similarly situated [...] called the dividing surface“.

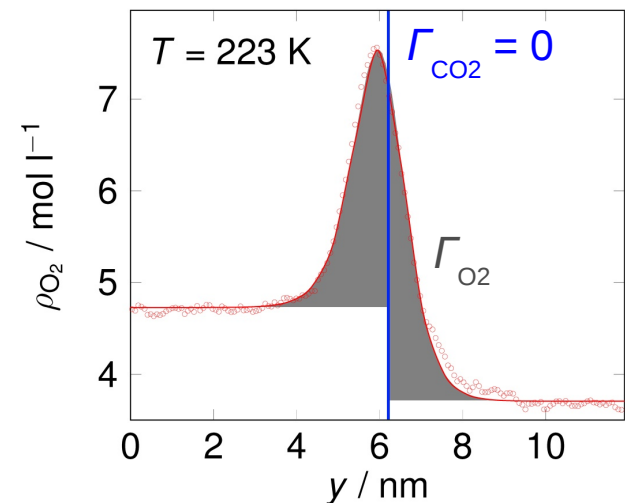
Thermodynamic **excess quantities** are ascribed to this dividing surface

$$X^{\text{System}} = X^I + X^{II} + X^E,$$

e.g. the surface free energy or the **adsorption**.

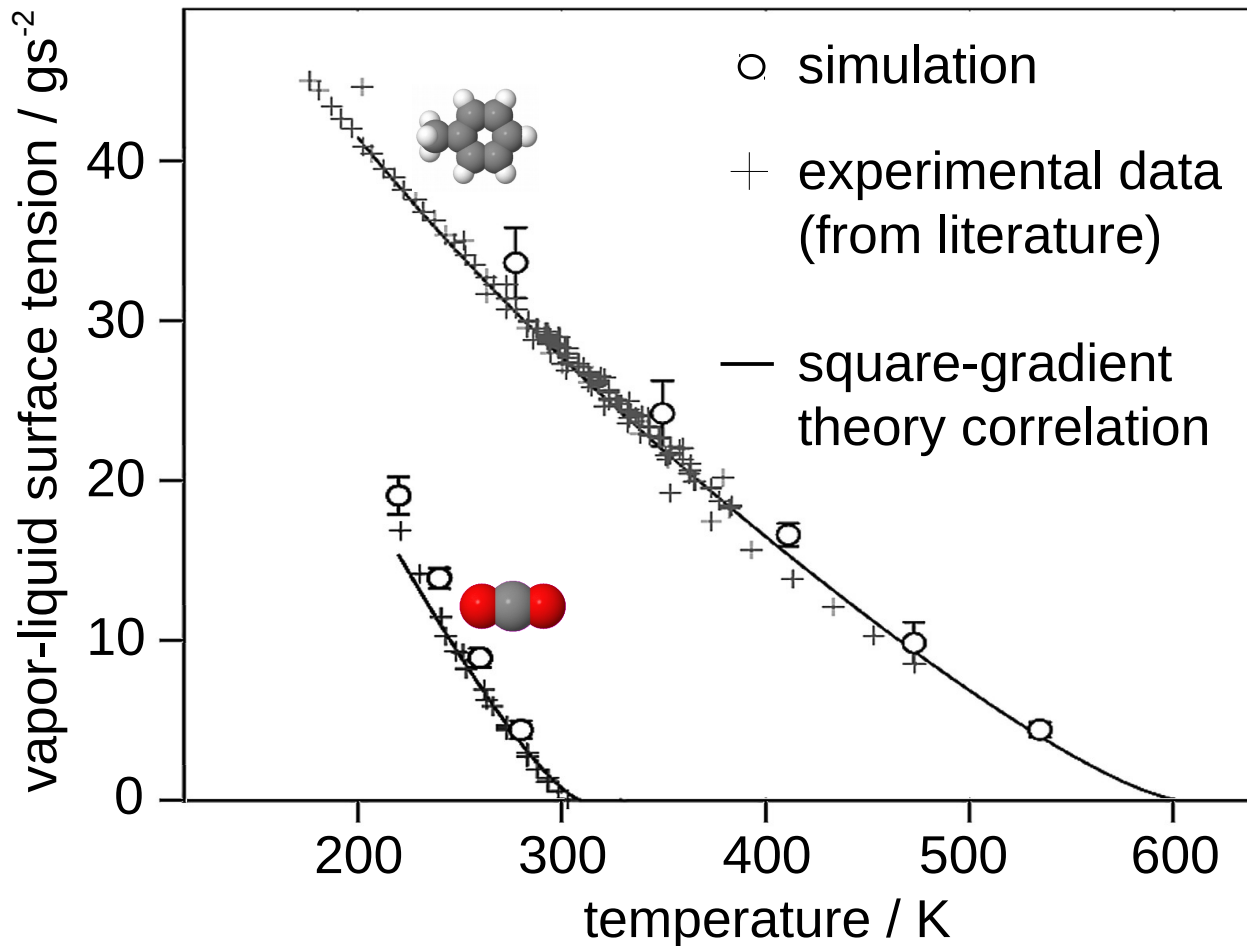
Beside absolute excesses, there are differential quantities, such as the surface tension

$$\gamma = \left( \frac{\partial F}{\partial A} \right)_{N, V, T}.$$

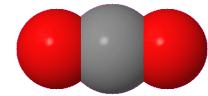




# Case study: Carbon dioxide and toluene

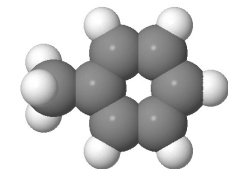


CO<sub>2</sub>



3CLJQ  
(Merker *et al.*)

**Toluene**



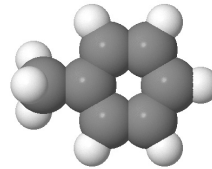
7CLJD+5Q  
(Huang *et al.*)

Adjusted to  
pure compo-  
nent VLE data



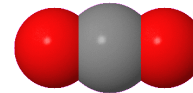
# Interfacial enrichment of carrier gases

Toluene

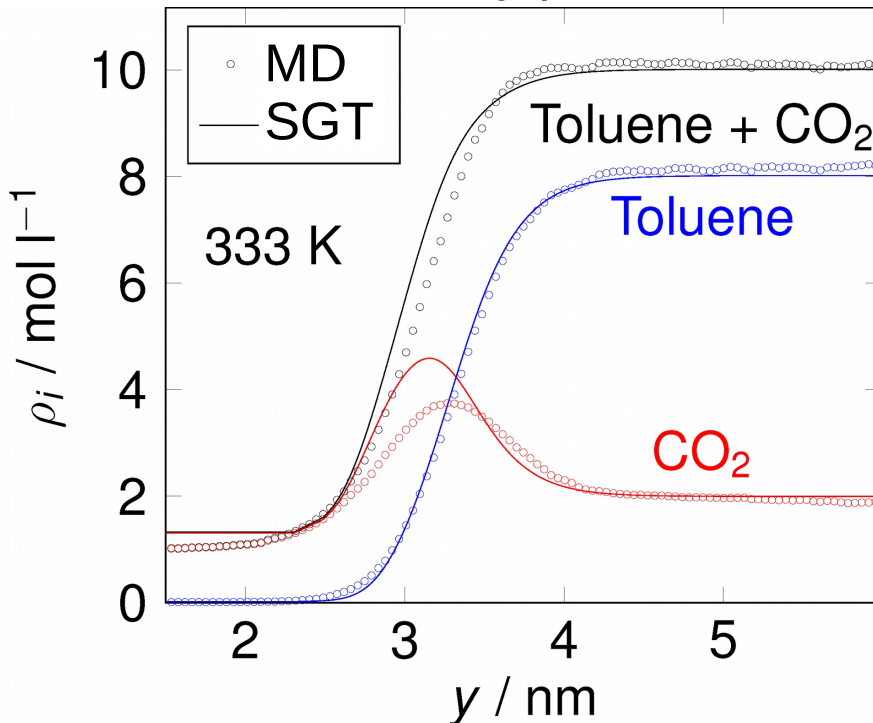


+

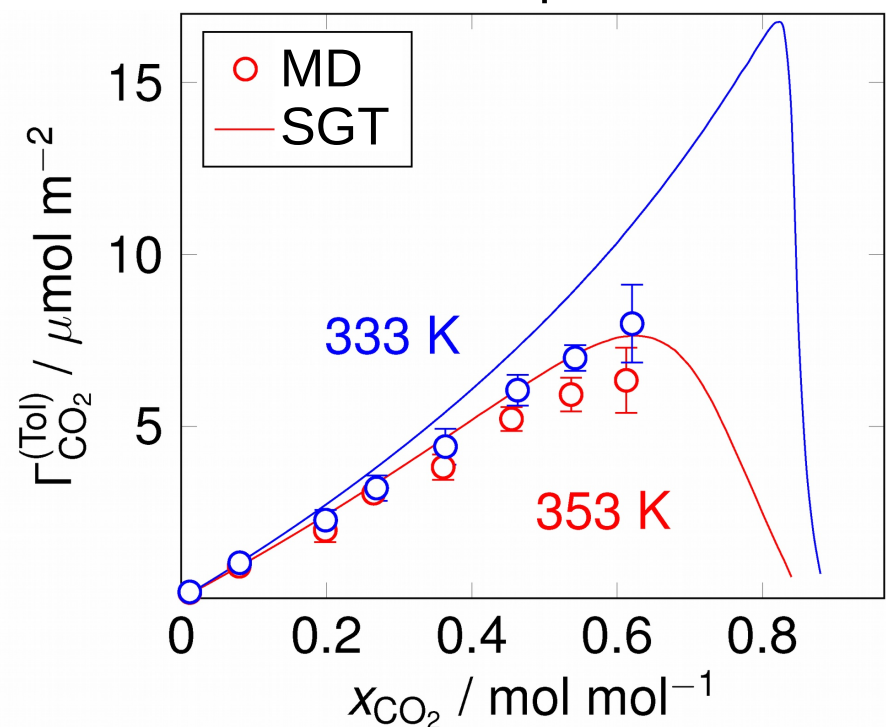
CO<sub>2</sub>



density profile



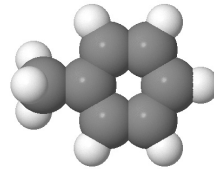
adsorption





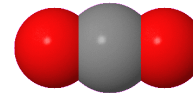
# Adsorption effect on the surface tension

Toluene

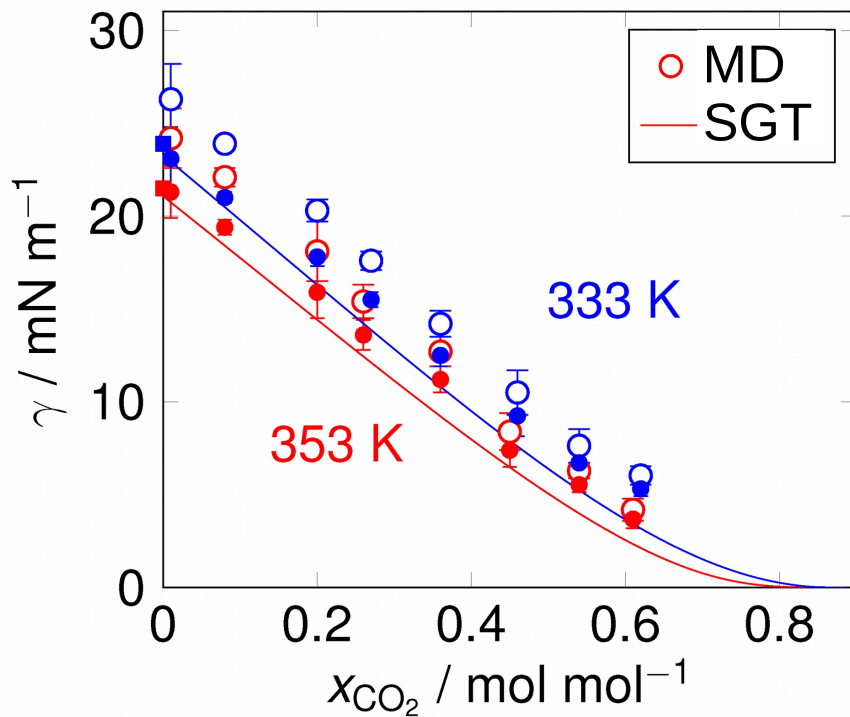


+

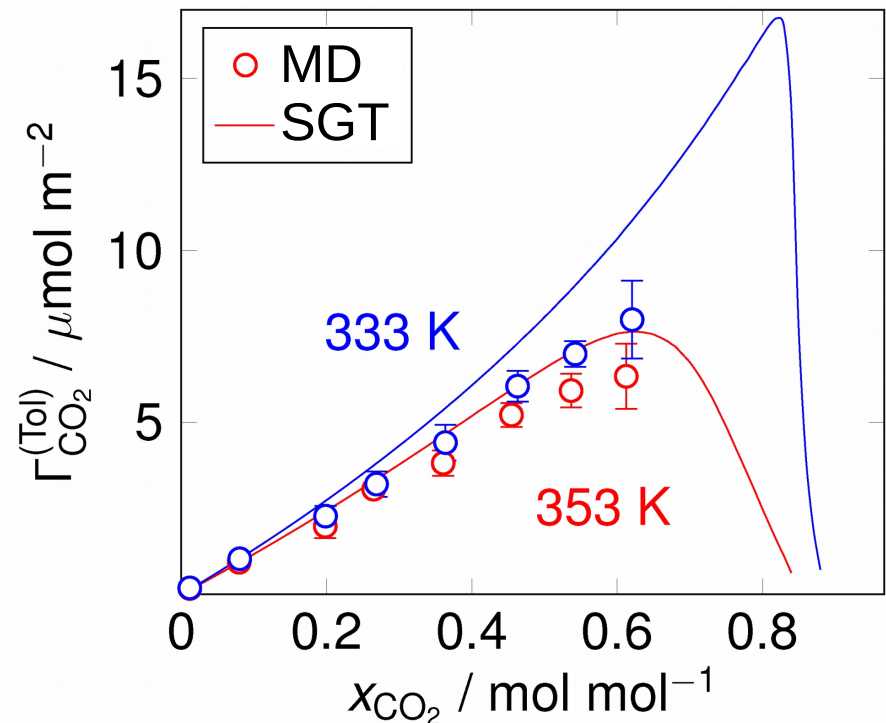
CO<sub>2</sub>



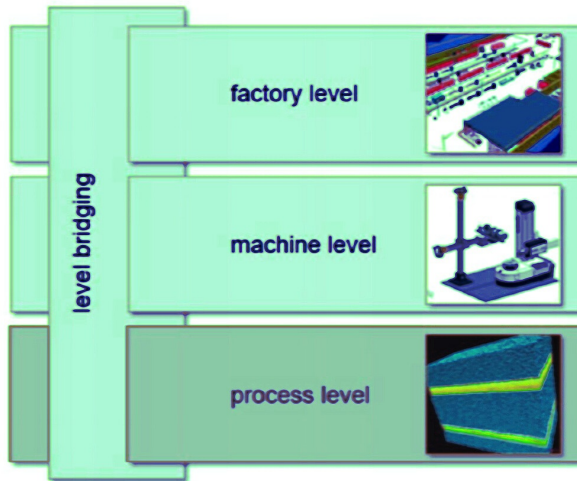
surface tension



adsorption



# International Graduate School IRTG 2057



Projects of the German partners are funded by the German Research Foundation (DFG).

Focus: Exchange of doctoral candidates.

**Contacts at UC Berkeley  
(Mechanical Engineering)**

David Dornfeld  
Tarek I. Zohdi

**Contact at Berkeley Lab**

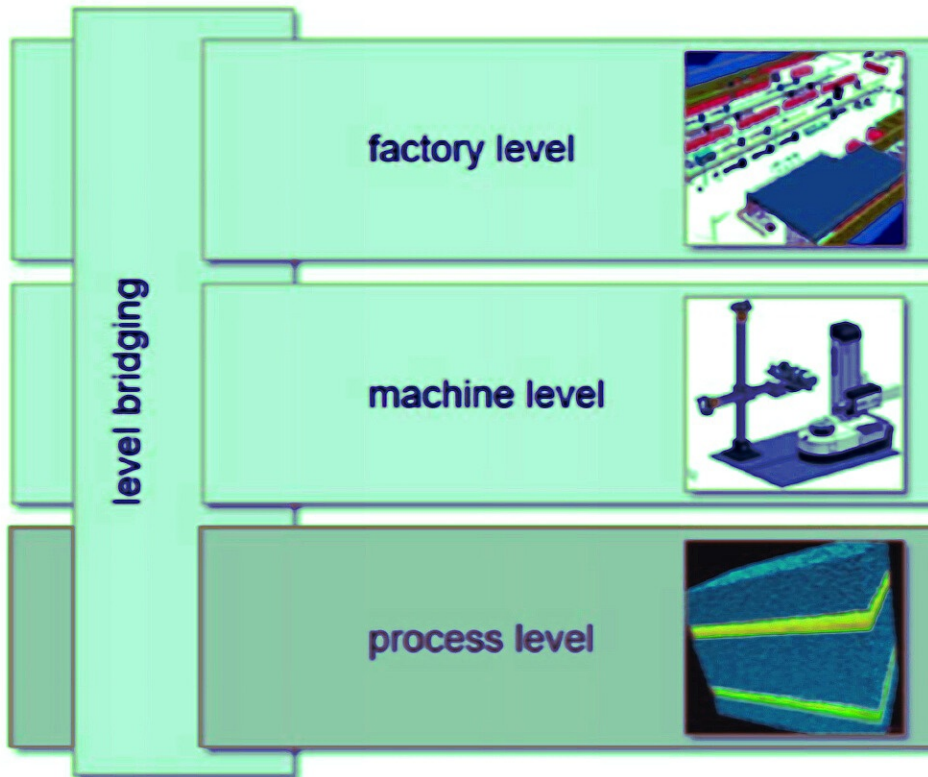
Gunther H. Weber

**IRTG2057**  
Physical Modeling for  
Virtual Manufacturing Systems and Processes





# International Graduate School IRTG 2057



Aim of our subproject:

Investigate elementary processes and properties that are relevant for understanding the role of cutting liquids during machining.

Method: MD simulation.





# International Graduate School IRTG 2057



*Martin  
Lautenschläger*



*Simon Stephan*  
(from January 2016)

Aim of our subproject:

Investigate elementary processes and properties that are relevant for understanding the role of cutting liquids during machining.

Method: MD simulation.



# Summary

## The traditional art of molecular modeling

An **expert modelling artist** designs and publishes

- a single optimized model for a particular fluid,
- according to his choice of criteria (often unknown to the public),
- users are passive, they have to live with the artists' decision.

## Scientific modeling by multicriteria optimization

For established model classes and multiple thermodynamic criteria,

- the dependence of thermodynamic properties on the model parameters is determined and correlated,
- the deviation between model properties and real fluid behaviour is characterized, and the Pareto set is published,
- users can design their own tailored model with **minimal effort**.



# Summary

With optimized and validated **molecular models**, e.g. from multicriteria optimization, quantitatively reliable predictions can be made.

Massively parallel MD simulation of large systems makes activated processes like homogeneous nucleation directly accessible. In this way, **over 100 000 cores** of a supercomputer can be used efficiently.

For **pure carbon dioxide**, homogeneous nucleation of bubbles in a metastable liquid and of droplets in a metastable vapor is well described by CNT, without the need for a curvature correction.

The influence of a carrier gas cannot be reduced to its presence in the vapor phase, due to the possibility of **interfacial enrichment**.