



# Quantitatively reliable **massively-parallel** **molecular modelling** and **simulation** of **vapour-liquid interfaces**

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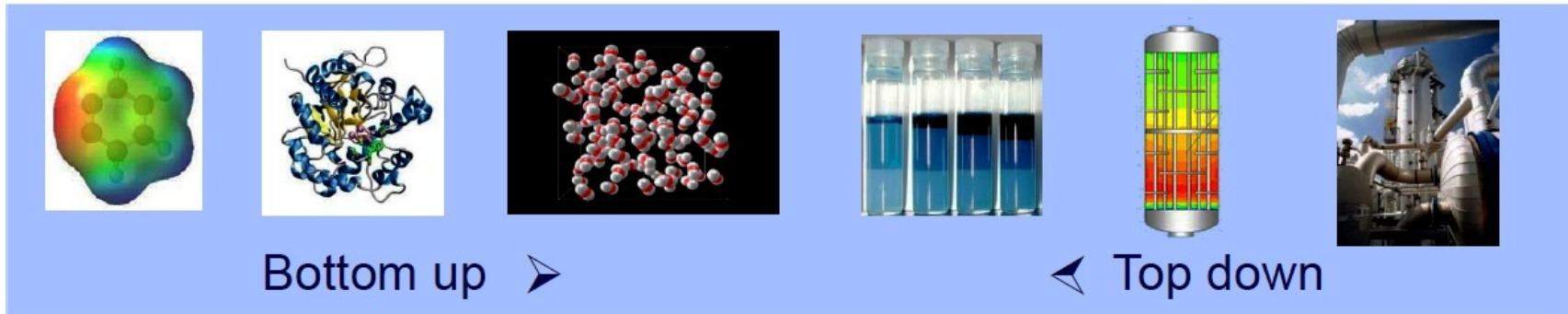


Kanpur, 30<sup>th</sup> December 2015  
**IITK Chemical Engineering Seminar**

**Computational  
Molecular Engineering**



# Computational molecular engineering



## From Physics (qualitative accuracy)

- Physically realistic modelling of intermolecular interactions
- Separate contributions due to repulsive and dispersive as well as electrostatic interactions

## To Engineering (quantitative reliability)

- No blind fitting, but parameters of *effective pair potentials* are adjusted to experimental data
- Physical realism facilitates reliable interpolation and extrapolation

# Force-field models of low-molecular fluids

## Geometry

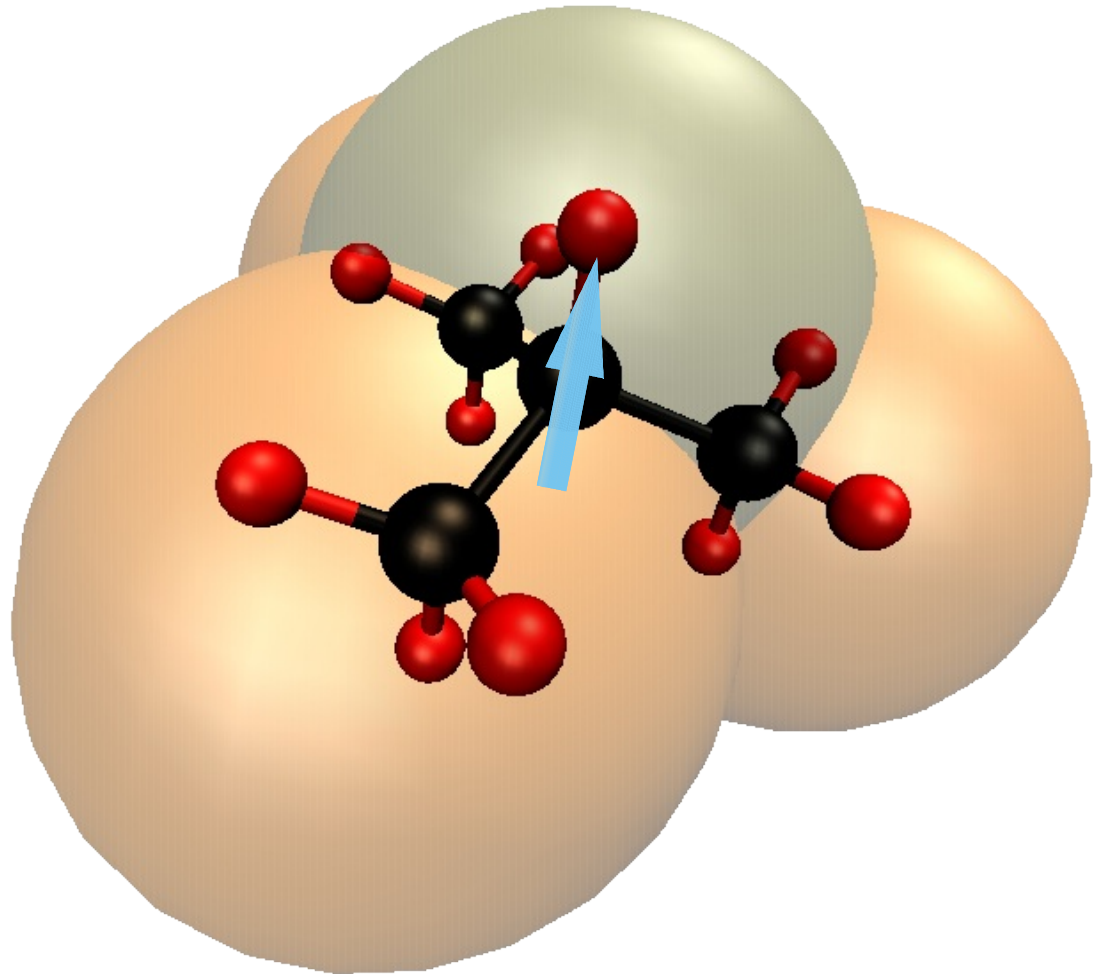
Bond lengths and angles

## Dispersion and repulsion

Lennard-Jones potential:  
Size and energy parameters

## Electrostatics

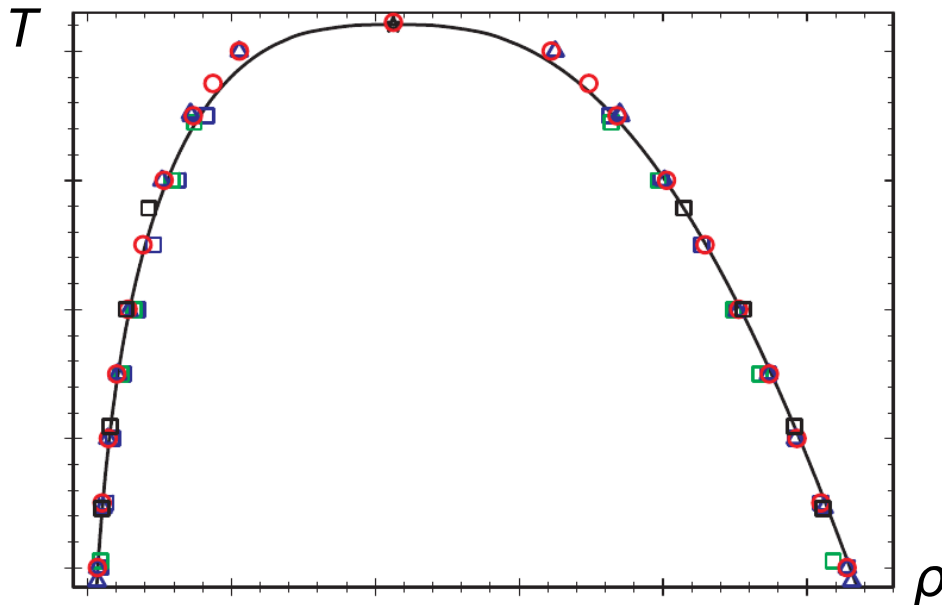
Point polarities  
(charge, dipole, quadrupole):  
Position and magnitude





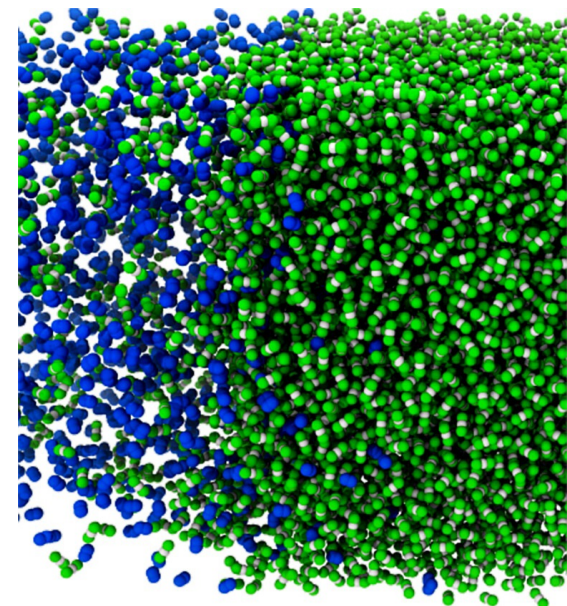
# Vapour-liquid equilibria

## Bulk properties



vapour pressure, saturated densities, composition, enthalpy of vaporization, etc., by Grand Equilibrium simulation

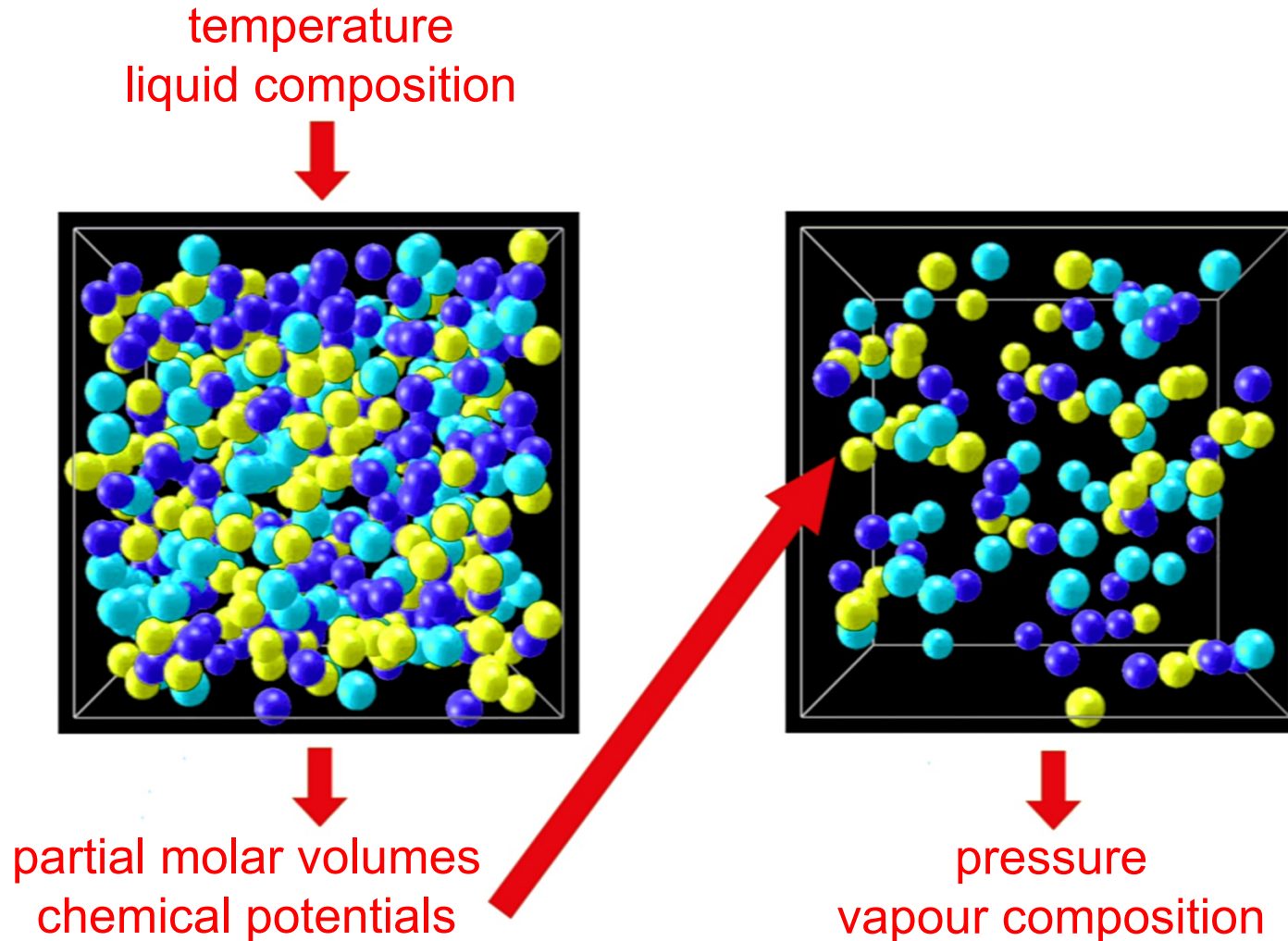
## Interfacial properties



heterogeneous systems with finite-size effects and long-range interactions

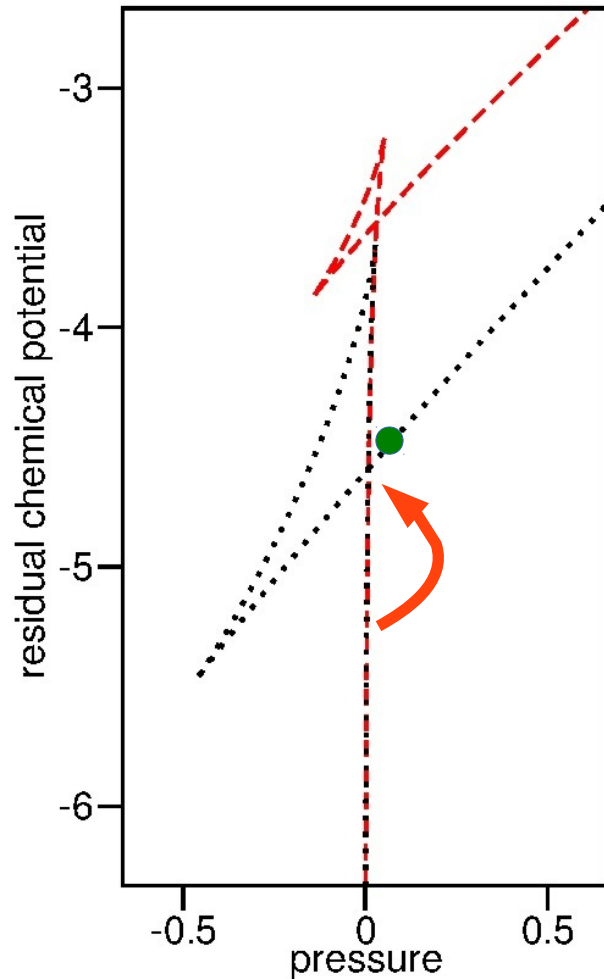


# Vapour-liquid equilibria: Grand equilibrium





# Vapour-liquid equilibria: Grand equilibrium



Given: Temperature  $T$ , liquid composition  $x$

First step:  $NpT$  simulation of the liquid phase

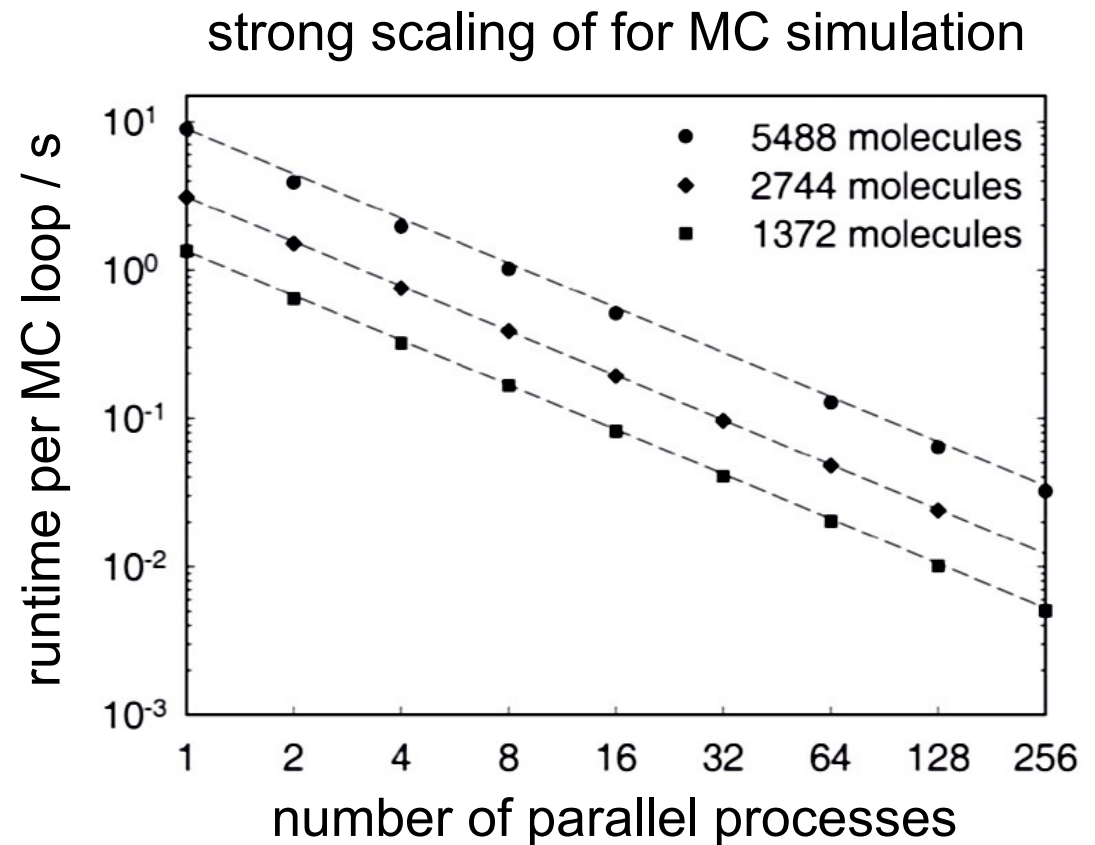
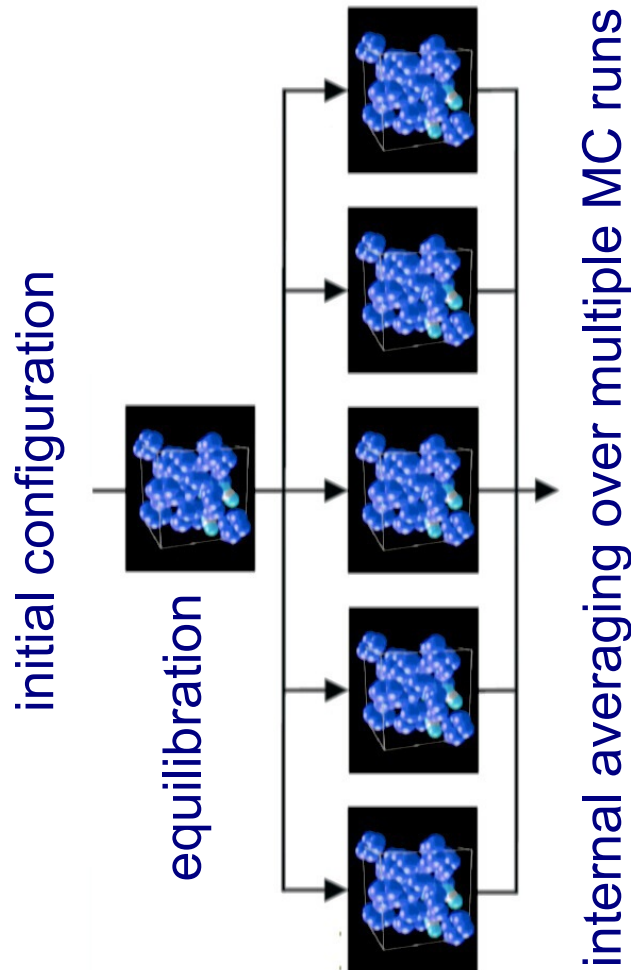
An estimate, which may deviate from  $p^{\text{sat}}(T)$ , is used for  $p$  in this simulation. The chemical potential and its first and second derivatives with respect to pressure are determined.

Second step: Pseudo- $\mu VT$  vapour simulation

Grand-canonical simulation where the value of  $\mu$  is determined on the fly from the pressure.

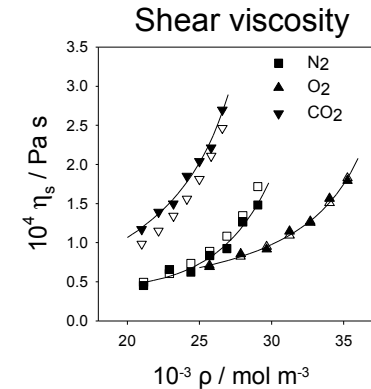
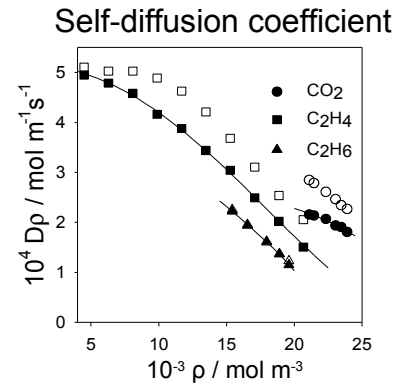
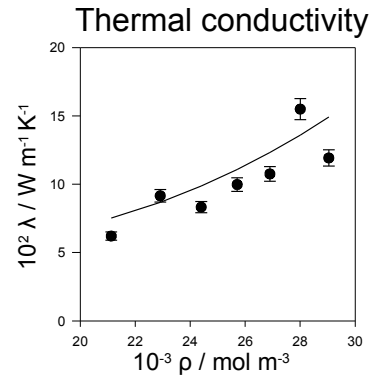
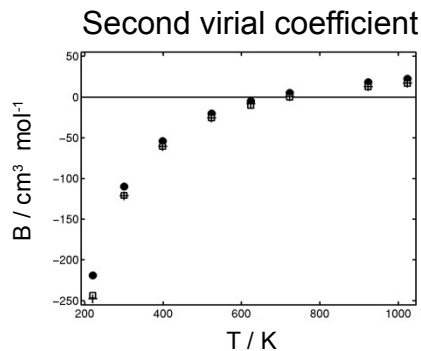
Obtained: Pressure  $p$ , vapour composition  $y$

# Concurrency of multiple Markov chains



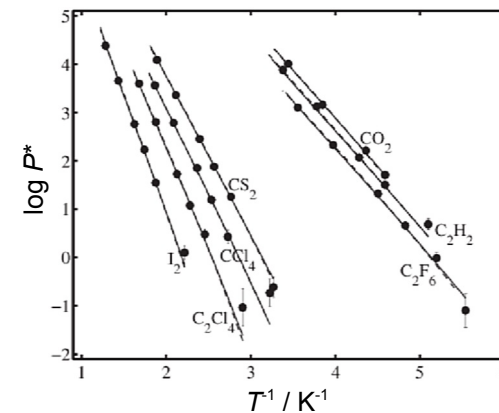
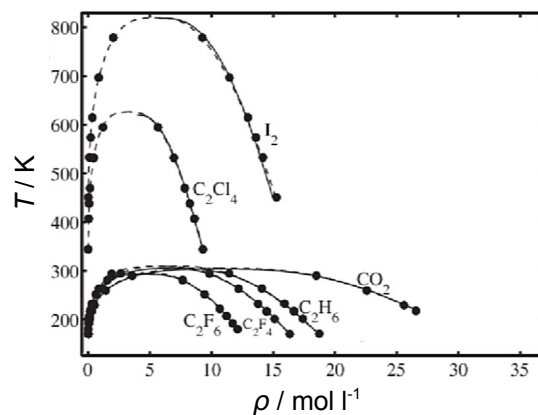


# Thermodynamic properties of bulk fluids



*ms2* is freely available for academic use: register at [www.ms-2.de](http://www.ms-2.de)

Vapour-liquid equilibria: Saturated densities and vapour pressures

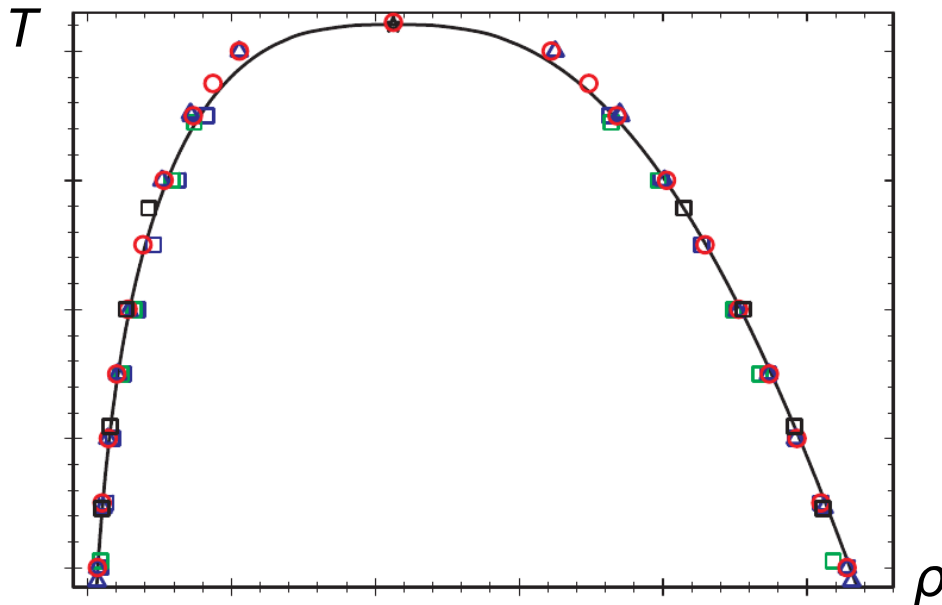






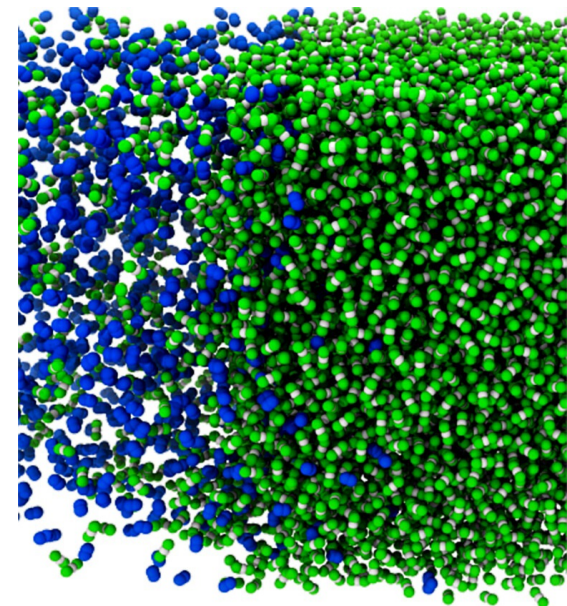
# Vapour-liquid interfaces

## Bulk properties



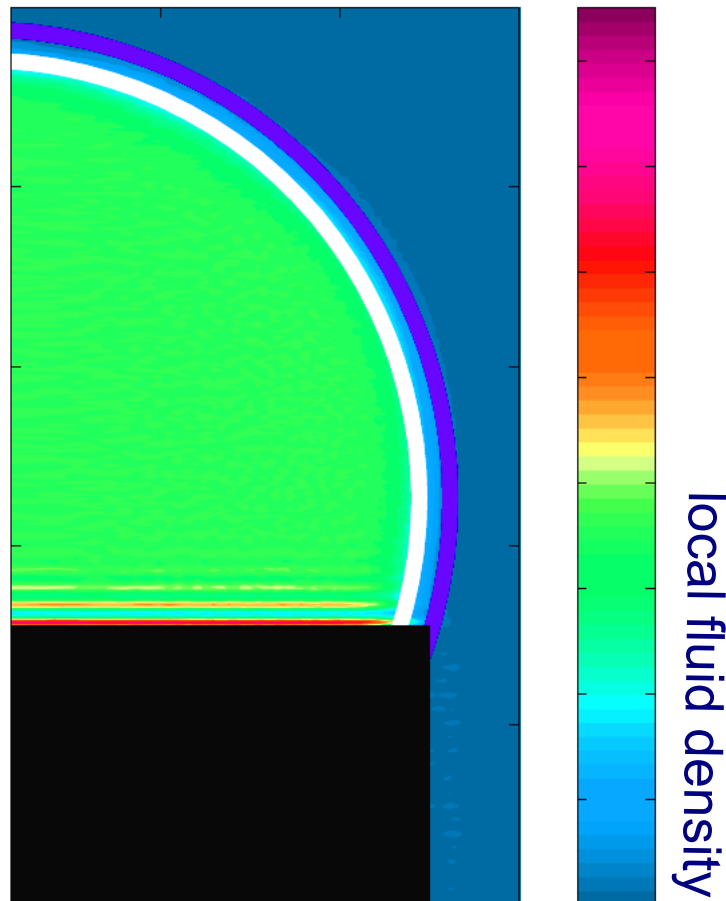
vapour pressure, saturated densities,  
 composition, enthalpy of vaporization,  
 etc., by Grand Equilibrium simulation

## Interfacial properties

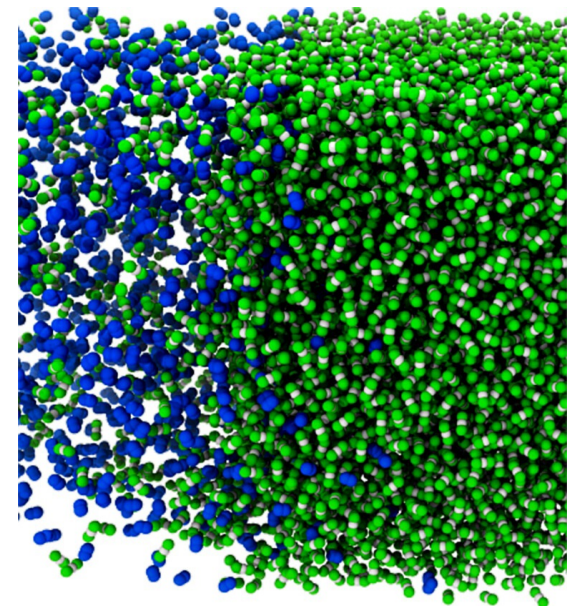


heterogeneous systems  
 with finite-size effects and  
 long-range interactions

# Molecular dynamics of fluids at interfaces



## Interfacial properties



heterogeneous systems  
with finite-size effects and  
long-range interactions

# Molecular dynamics of large systems



The screenshot shows the homepage of the Is1 Mardyn website. The header features the text "Is1 Mardyn" next to a large image of blue and white spheres representing a molecular simulation. Below the header is a navigation menu with options: Home, About (selected), Publications, Download, License, and Contact. The main content area is titled "About Is1 mardyn" and contains the following text:

The development of *Is1 mardyn* is jointly driven by

- [High Performance Computing Center Stuttgart \(HLRS\)](#), University of Stuttgart,
- [Laboratory for Engineering Thermodynamics \(LTD\)](#), University of Kaiserslautern,
- [Scientific Computing in Computer Science \(SCCS\)](#), Technische Universität München,
- [Thermodynamics and Energy Technology \(ThEt\)](#), University of Paderborn,

under the auspices of the Boltzmann-Zuse Society for Computational Molecular Engineering (BZS).

Please cite the work of Niethammer *et al.* (2014), *J. Chem. Theory Comput.* 10: 4455, in all publications containing the results of MD simulations with the *Is1 mardyn* program.

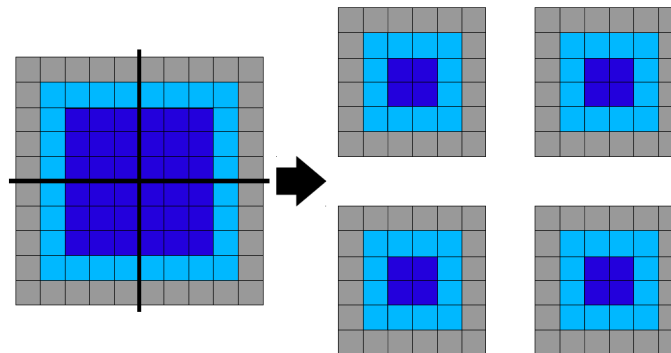
The development team can be contacted via the *Is1 mardyn* [contact point](#) at the University of Kaiserslautern.

At the bottom of the screenshot, the URL <http://www.is1-mardyn.de/> is displayed in a large white font on a black background.



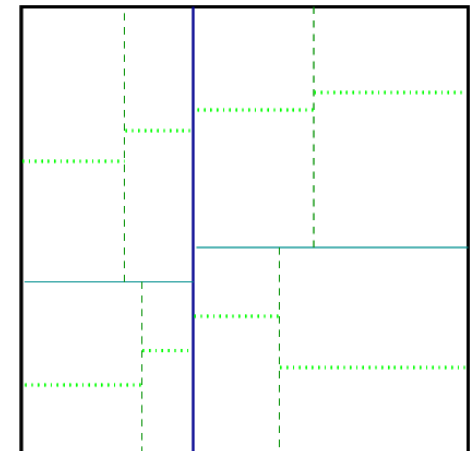
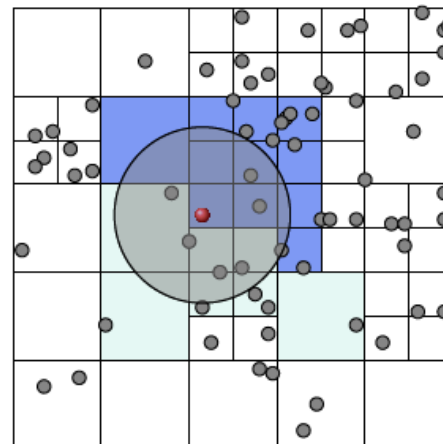
# Parallelization by volume decomposition

Linked-cell data structure suitable for spatial domain decomposition:



(non-blocking, overlapping MPI send/receive operations)

Methods for heterogeneous or fluctuating particle distributions:



large systems “1”: molecular dynamics

<http://www.ls1-mardyn.de/>





# Hyperthreading and vectorization

Memory-efficient implementation based on the linked-cell data structure:

21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

hyperthreaded sliding window

Optionally, forces acting on molecules are only stored until their cell leaves the sliding window.

Efficient vectorization:

- Optimization by hand, using advanced vector extensions (AVX).
- Conversion from array of structures (AoS) to structure of arrays (SoA).

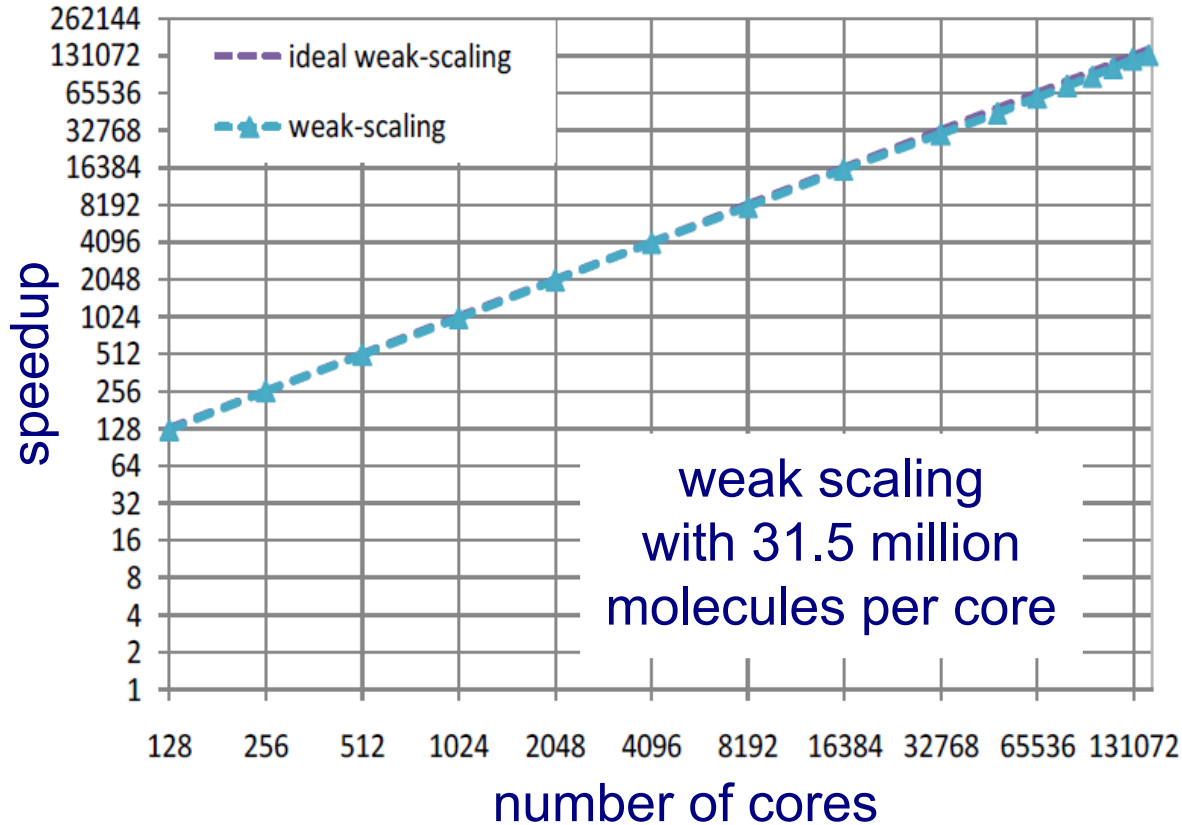
large systems “1”: molecular dynamics

<http://www.ls1-mardyn.de/>





# World record with Is1 mardyn on SuperMUC



Up to  $N = 4 \cdot 10^{12}$   
on SuperMUC



large systems "1": molecular dynamics

<http://www.ls1-mardyn.de/>



# MD simulation of homogeneous cavitation

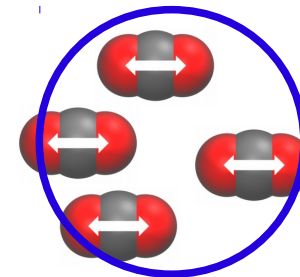
liquid CO<sub>2</sub> at 220 K and 22.6 mol/l



13 x 10<sup>6</sup> molecules (52 x 10<sup>6</sup> sites)

Scale-up to the entire *hermit* cluster for canonical simulation of cavitation in carbon dioxide.

Evaluation of local density at 180 x 180 x 180 grid points:



≤ 5 ?  
(6 mol/l)

Liquid phase detected for more than 5 neighbours within a radius of 6.9 Å around the grid point.

# MD simulation of homogeneous cavitation

liquid CO<sub>2</sub> at 220 K and 23.9 mol/l



Three consecutive regimes:

- relaxation (equilibration)
- homogeneous cavitation
- growth beyond critical size

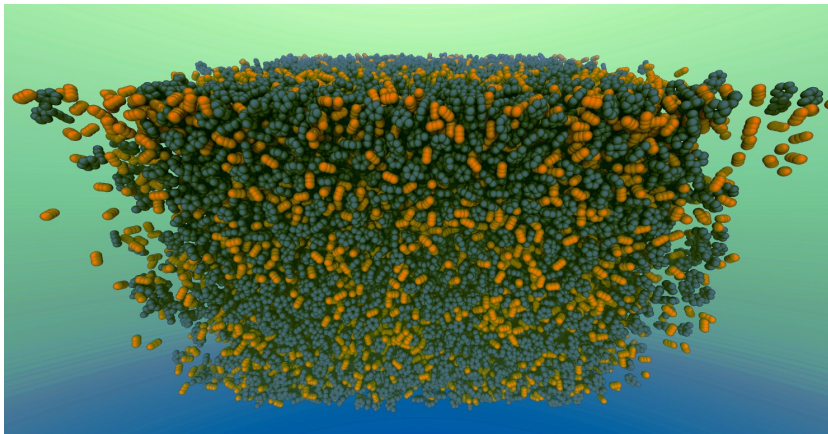
These and other simulations of interfacial phenomena crucially depend on an accurate surface tension.

$13 \times 10^6$  molecules ( $52 \times 10^6$  sites)

# Long-range correction at planar interfaces

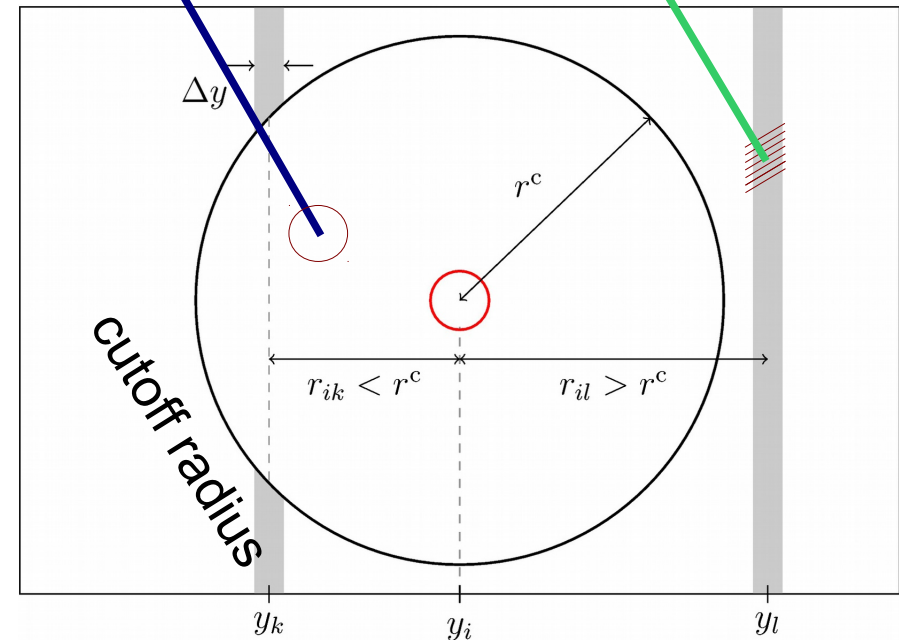
For planar interfaces:

**Long-range correction** from the density profile, following **Janeček**.



short range  
(explicit)

long range  
(correction)



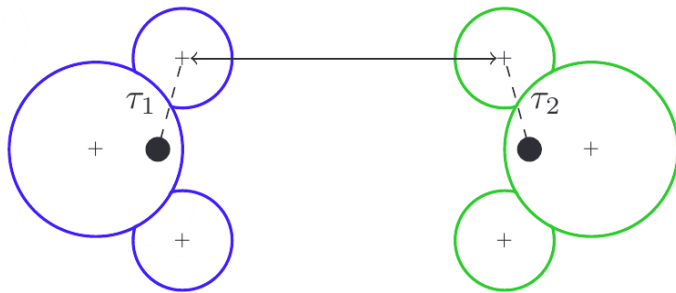
Full evaluation of all pairwise interactions is too expensive ...

... **short-range interactions** are evaluated only for **neighbours**.

# Long-range correction at planar interfaces

For planar interfaces:

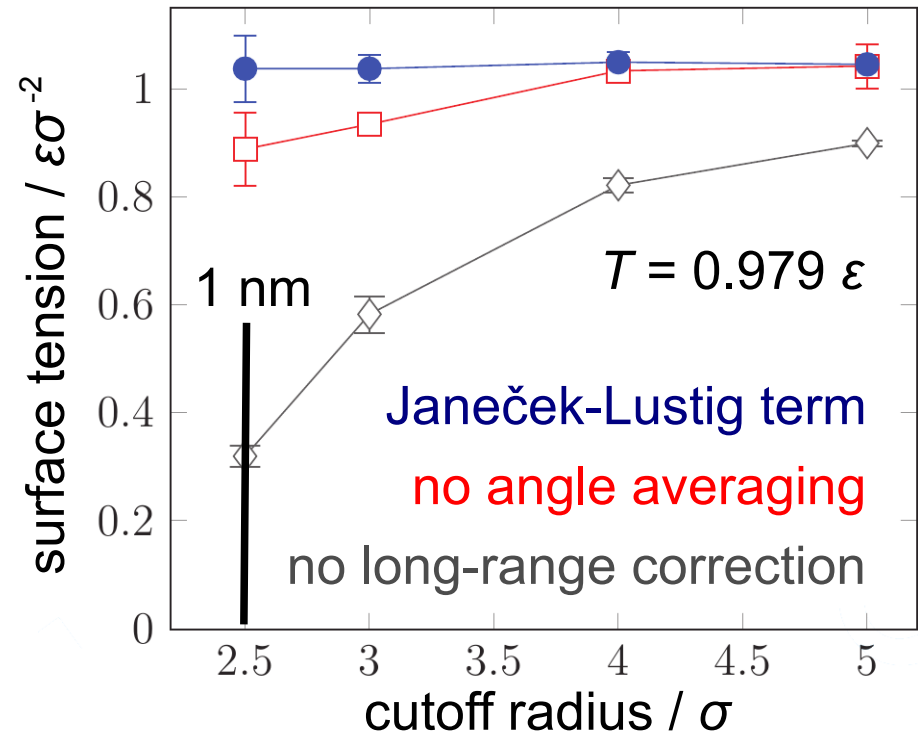
**Long-range correction** from the density profile, following **Janeček**.



**Angle-averaging** expression for multi-site models, following **Lustig**.

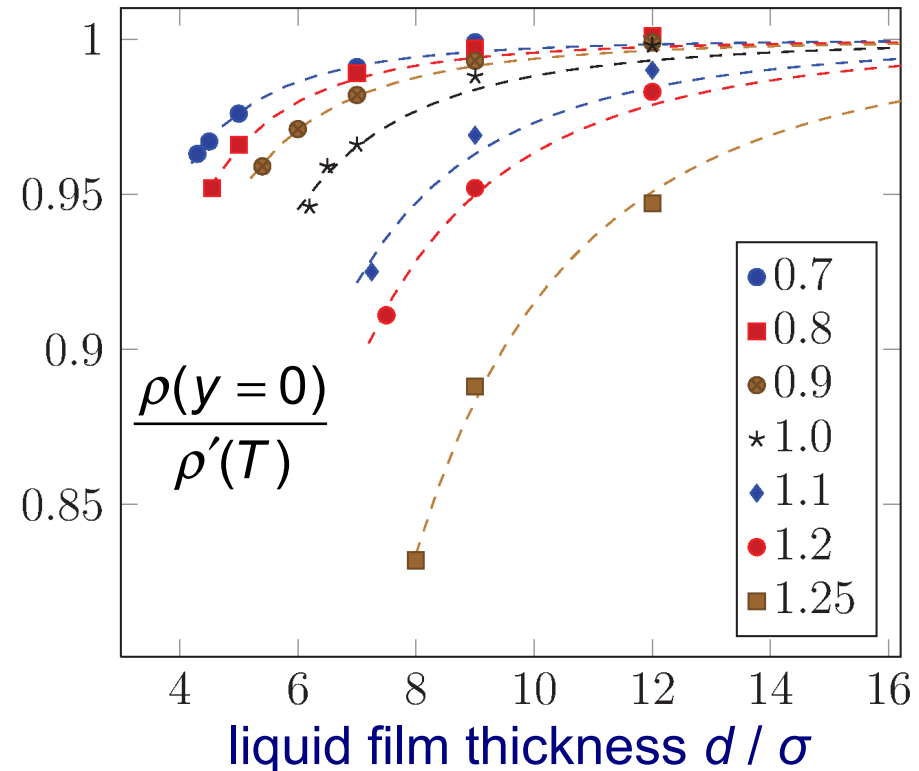
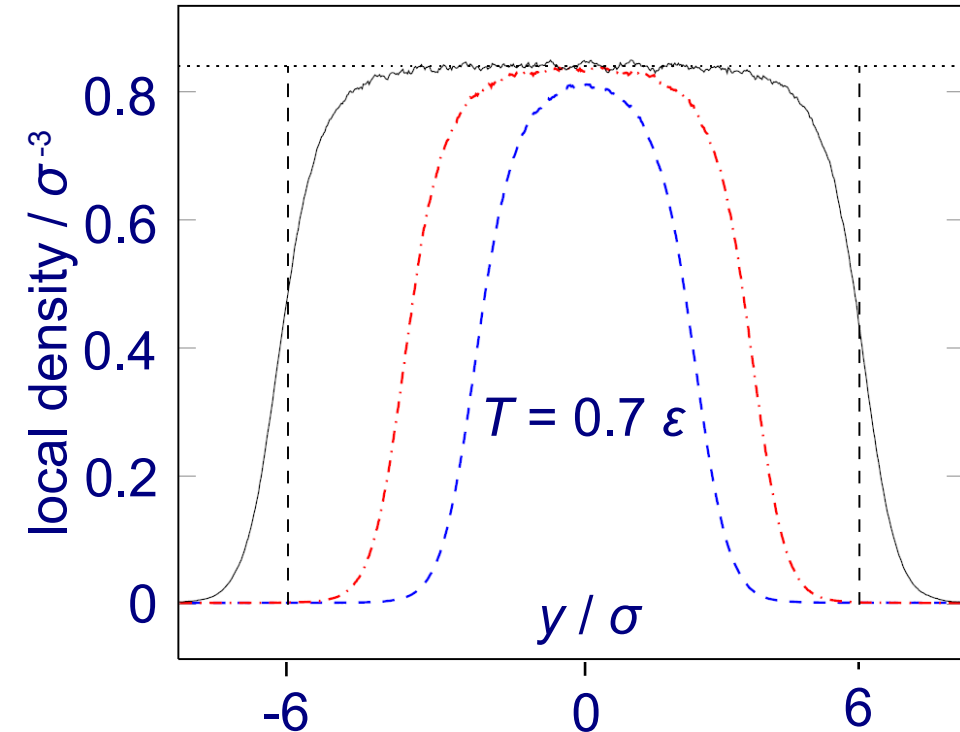
Dipole and dispersion lead to analogous long-range correction expressions. The long-range contribution of the quadrupole can be neglected.

Two-centre LJ fluid (2CLJ)





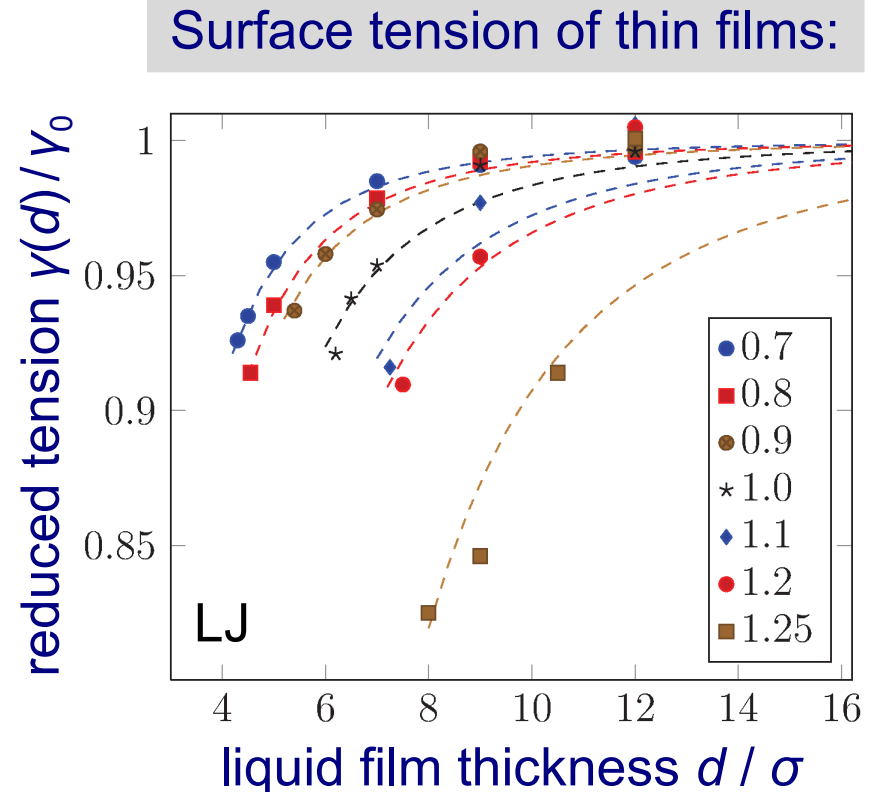
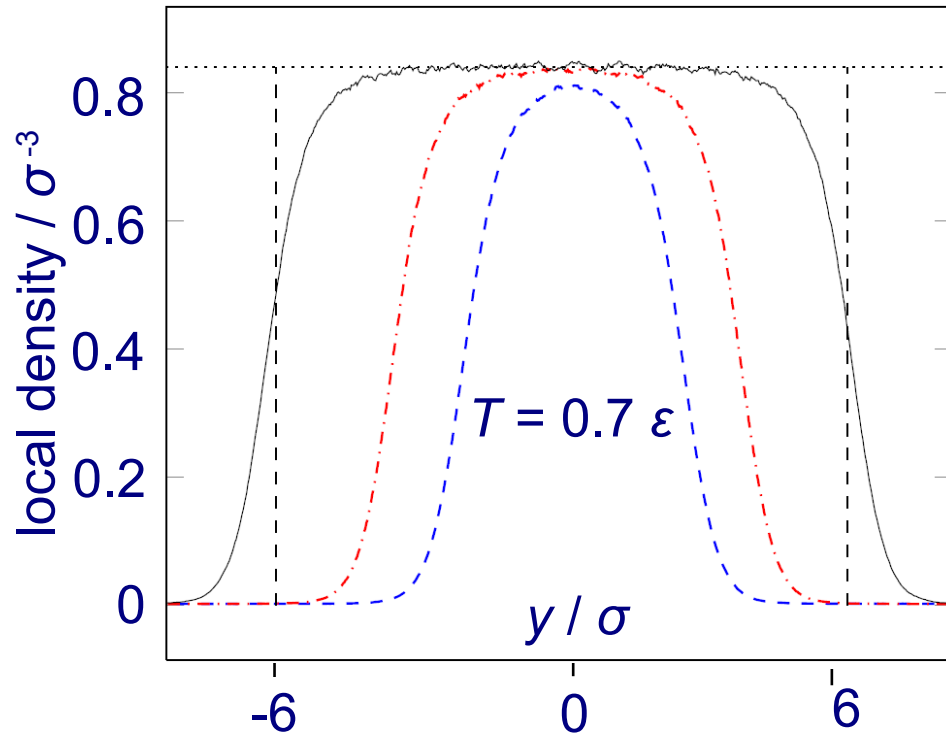
# Vapour-liquid interfaces: Finite-size effects



The density at the centre of a liquid nanofilm deviates from that of the saturated bulk liquid at the same temperature (scaling  $\sim 1/d^3$ ).



# Vapour-liquid interfaces: Finite-size effects

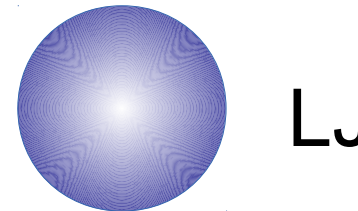
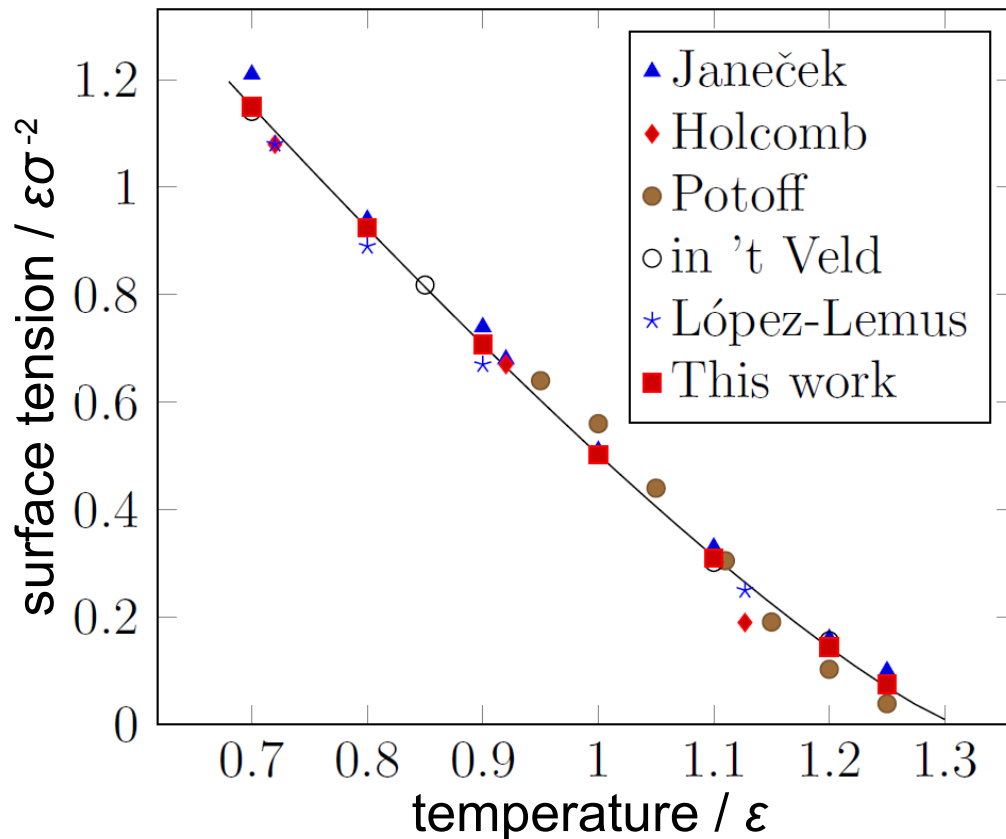


The deviation of the surface tension of a nanofilm from the macroscopic value exhibits the same tendency (scaling with  $1/d^3$ ).



# Surface tension at high precision

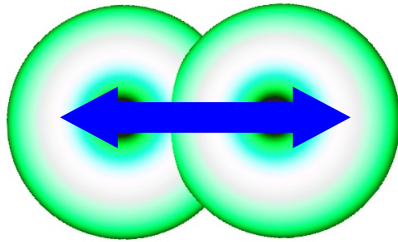
Lennard-Jones fluid



LJ

$$\gamma_0(T) = 2.94 \frac{\varepsilon}{\sigma^2} \left( 1 - \frac{T}{T_c} \right)^{1.23}$$

# Validation of molecular force field models



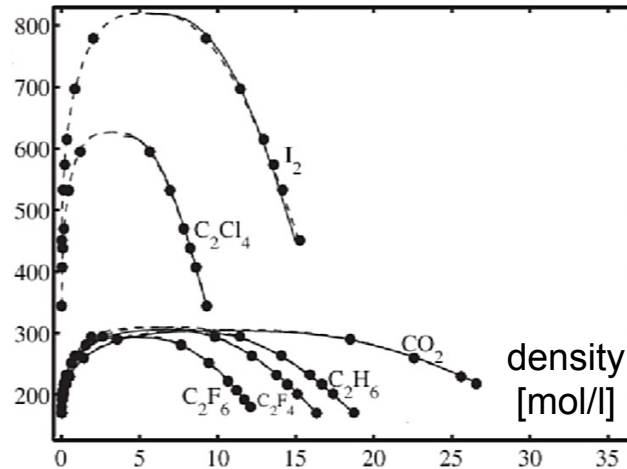
2CLJQ models:

- 2 LJ centres
- 1 quadrupole

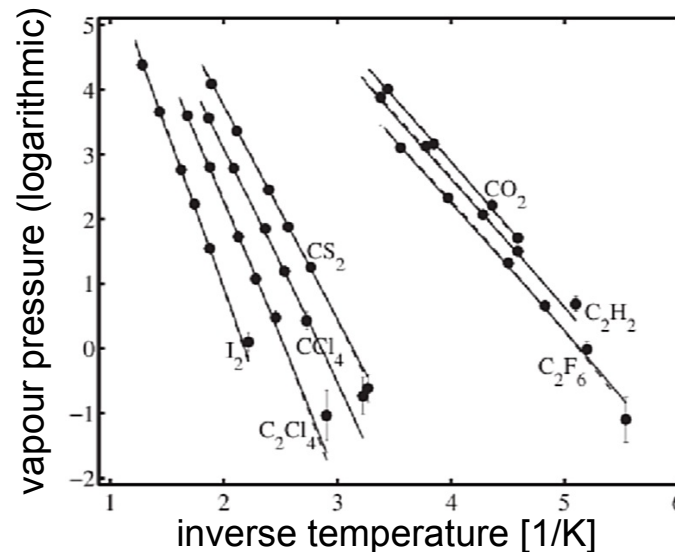
Fit of parameters  $\sigma$ ,  $\epsilon$ ,  
 $L$ ,  $Q$  to VLE data of  
29 fluids by Stoll *et al.*

Deviation:

- $\delta\rho' \approx 1\%$
- $\delta P^{\text{sat}} \approx 5\%$

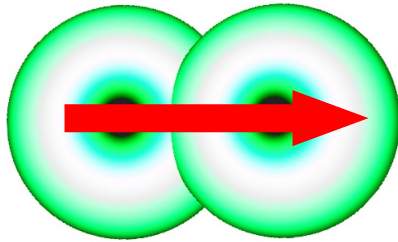


- simulation
- DIPPR correlation



No interfacial  
properties were  
considered for the  
parameterization.

# Validation of molecular force field models



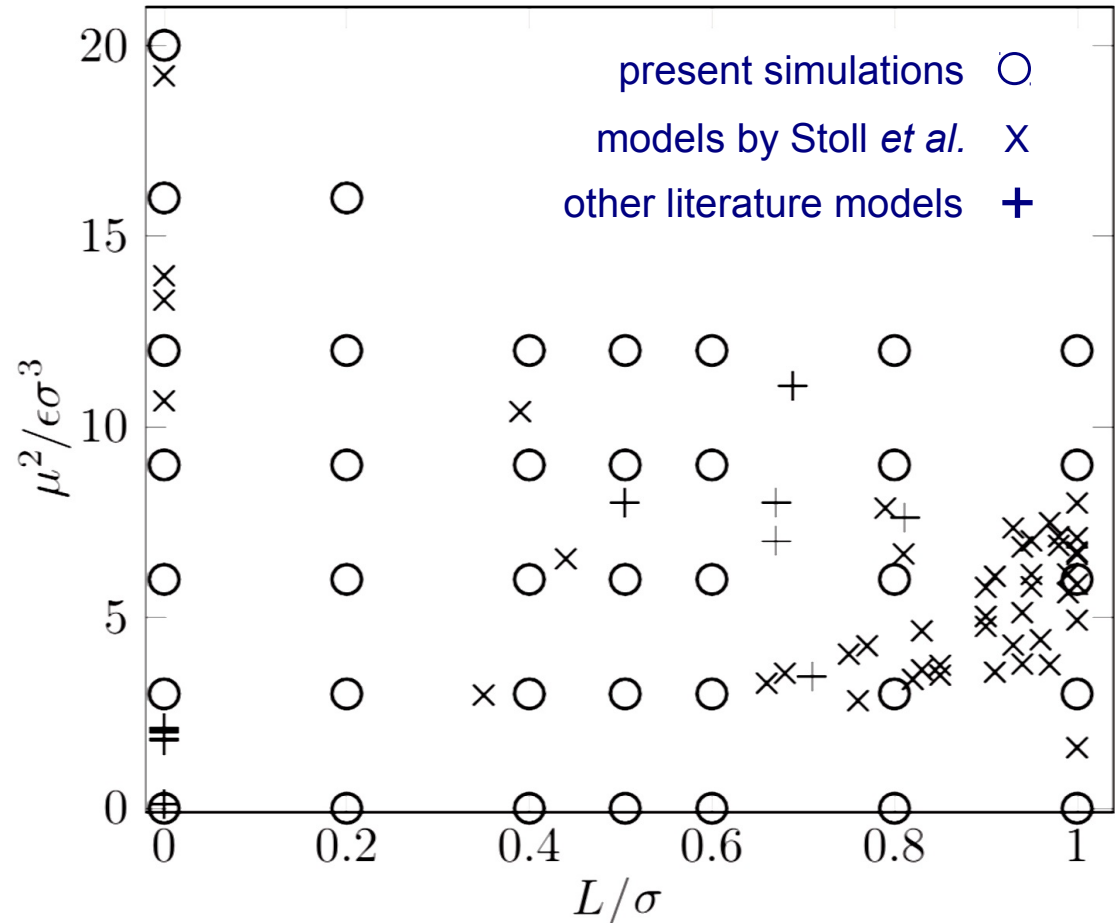
2CLJD models:

- 2 LJ centres
- 1 dipole

Fit of parameters  $\sigma$ ,  $\epsilon$ ,  
 $L$ ,  $\mu$  to VLE data of  
46 fluids by Stoll *et al.*

Deviation:

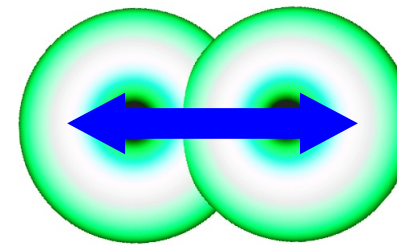
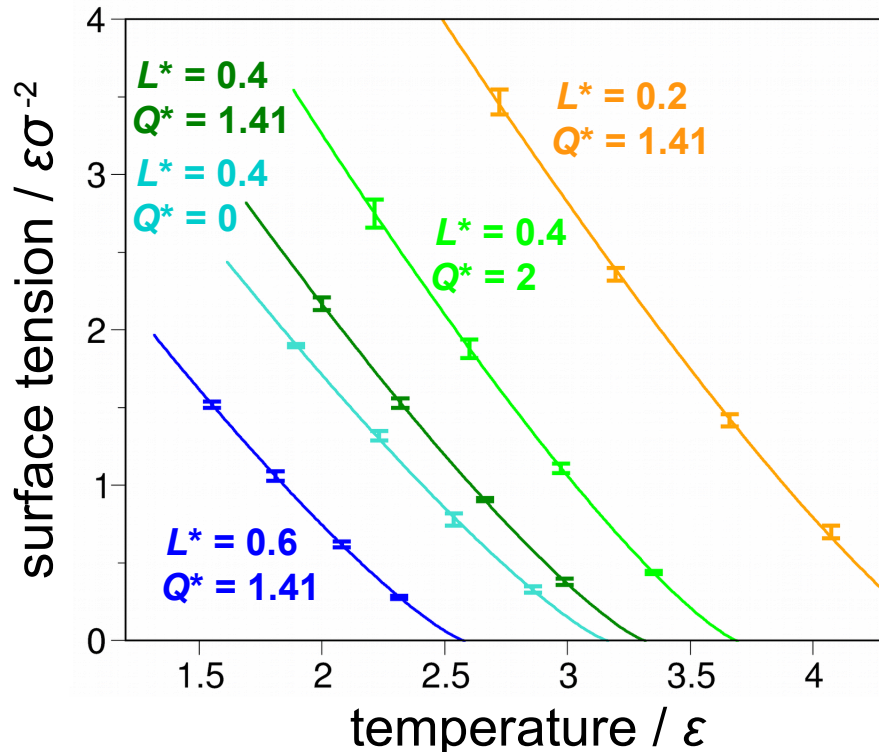
- $\delta\rho' \approx 1\%$
- $\delta P^{\text{sat}} \approx 5\%$





# Massively parallel molecular modelling

Two LJ + quadrupole (2CLJQ)



2CLJQ

Model parameters:

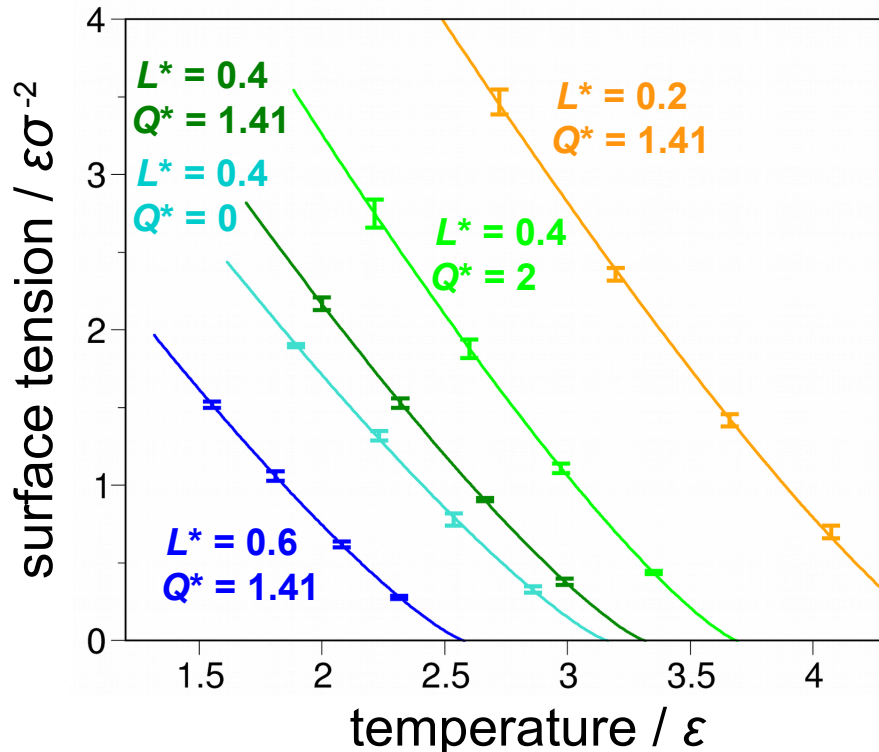
- LJ size parameter  $\sigma$
- LJ energy parameter  $\epsilon$
- Elongation  $L$
- Quadrupole moment  $Q$

- Systematic exploration of the four-dimensional model parameter space

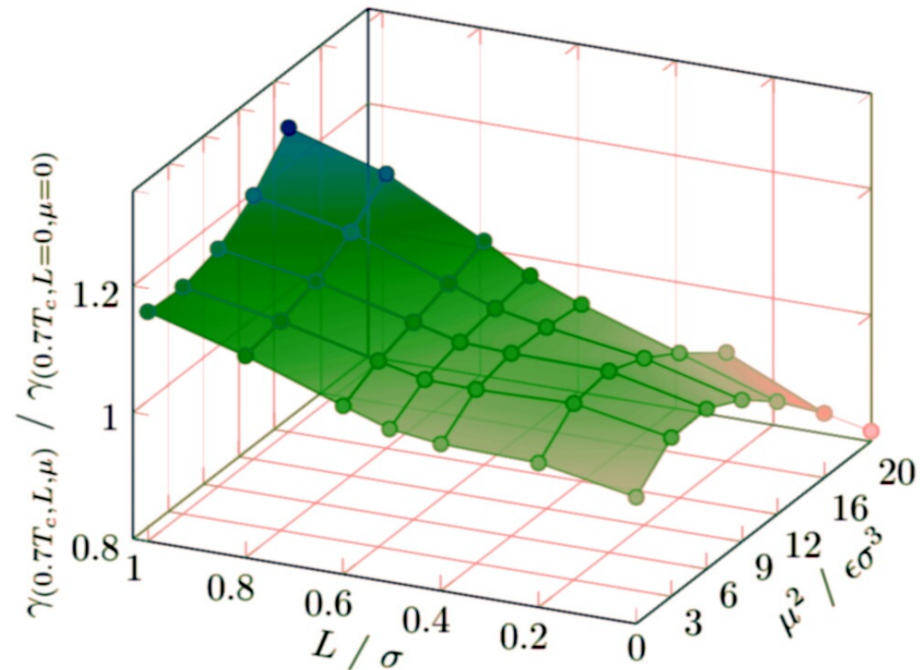


# Massively parallel molecular modelling

Two LJ + quadrupole (2CLJQ)



Two LJ + dipole (2CLJD)

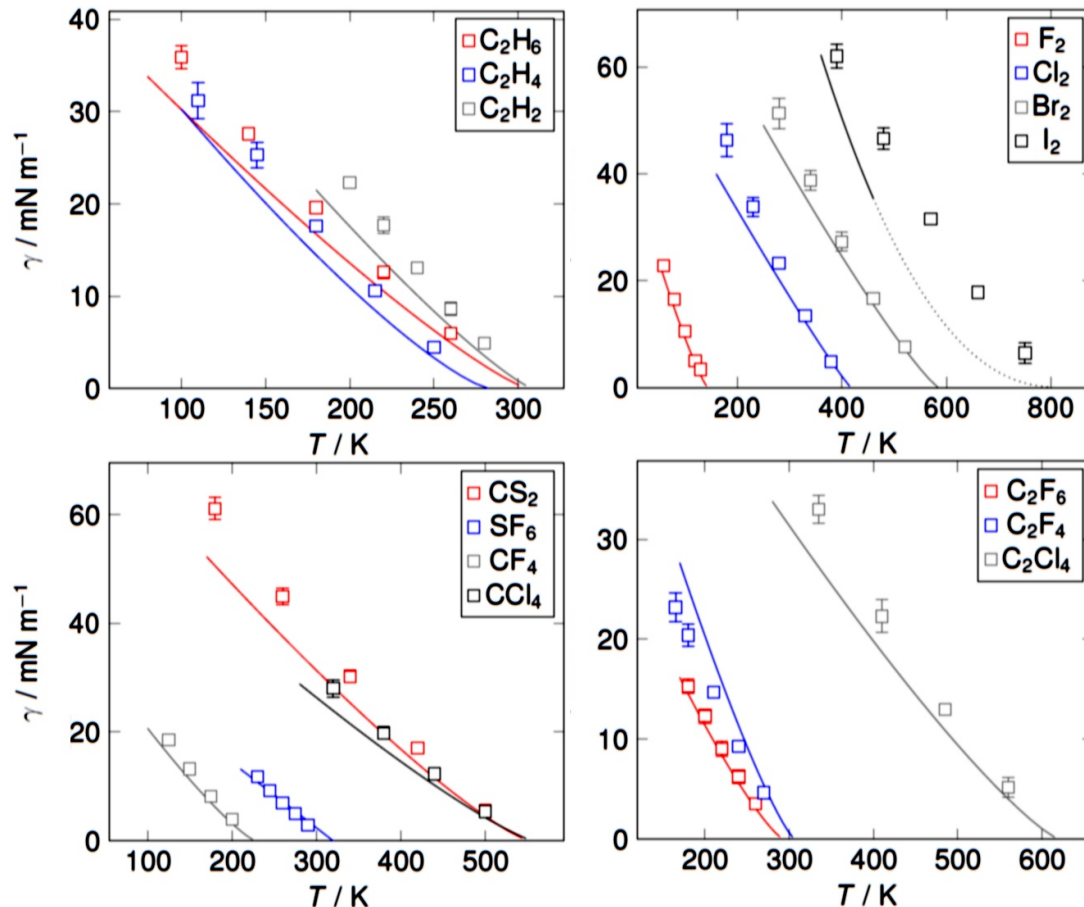


- Systematic exploration of the four-dimensional model parameter space
- Correlation of the surface tension by a critical scaling expression

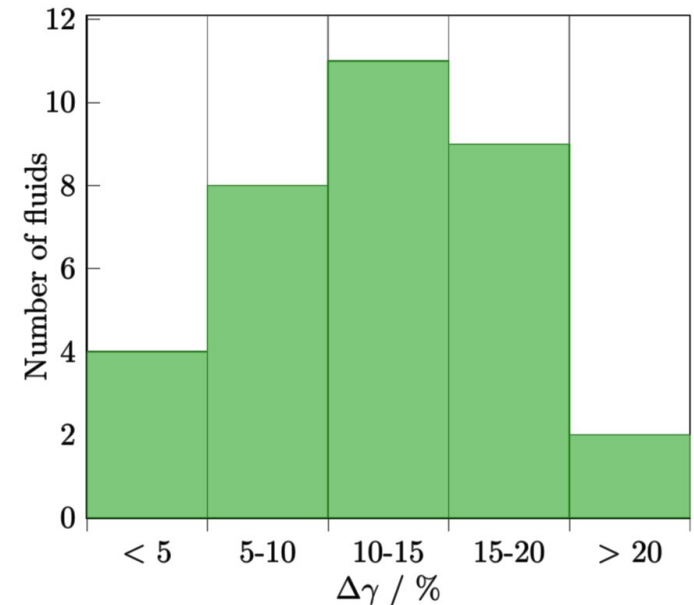



# Validation of molecular force field models

Two LJ + quadrupole (2CLJQ)



Two LJ + dipole (2CLJD)

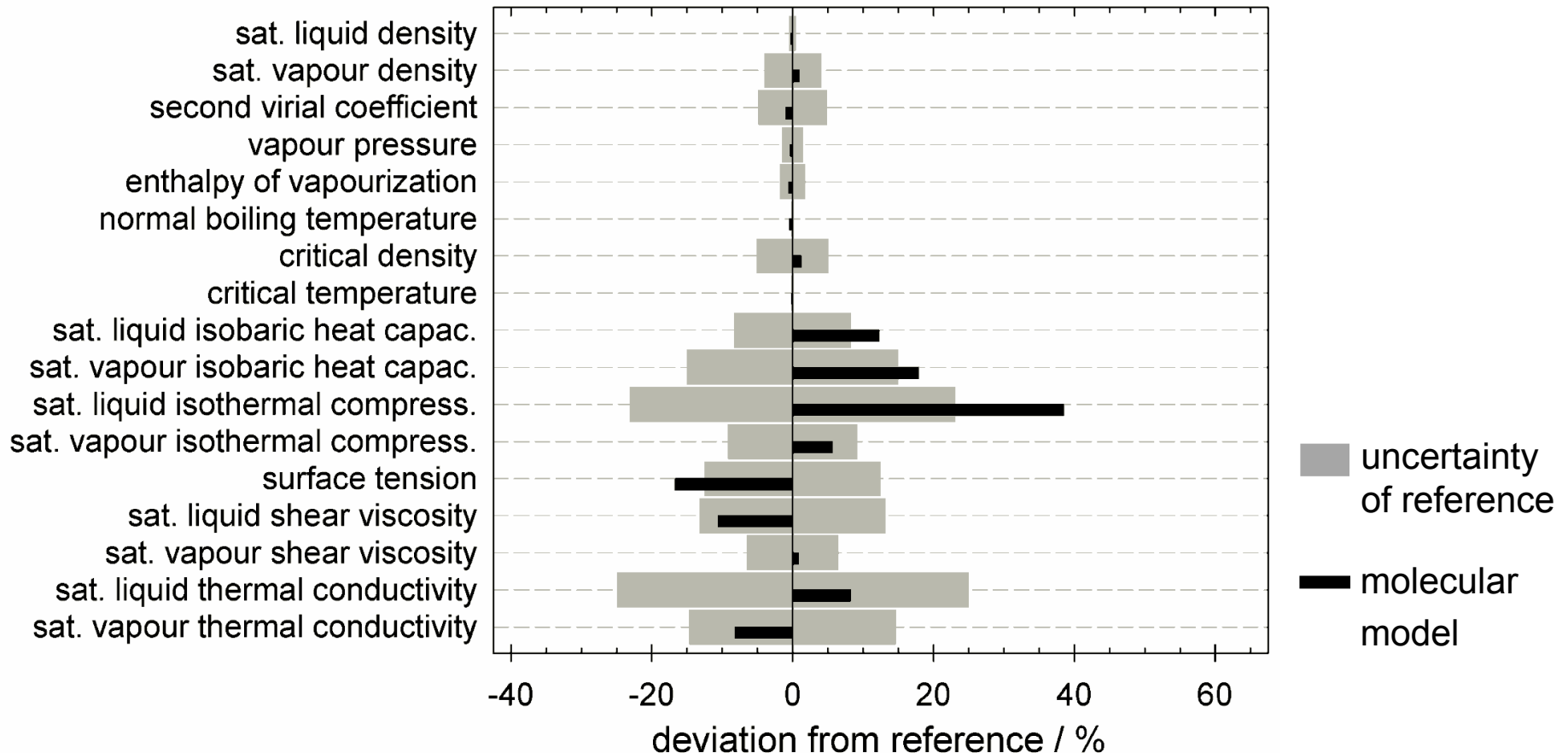


Fit to bulk properties   
10 to 20 % overestimation of  
vapour-liquid surface tension



# Model optimization with multiple objectives

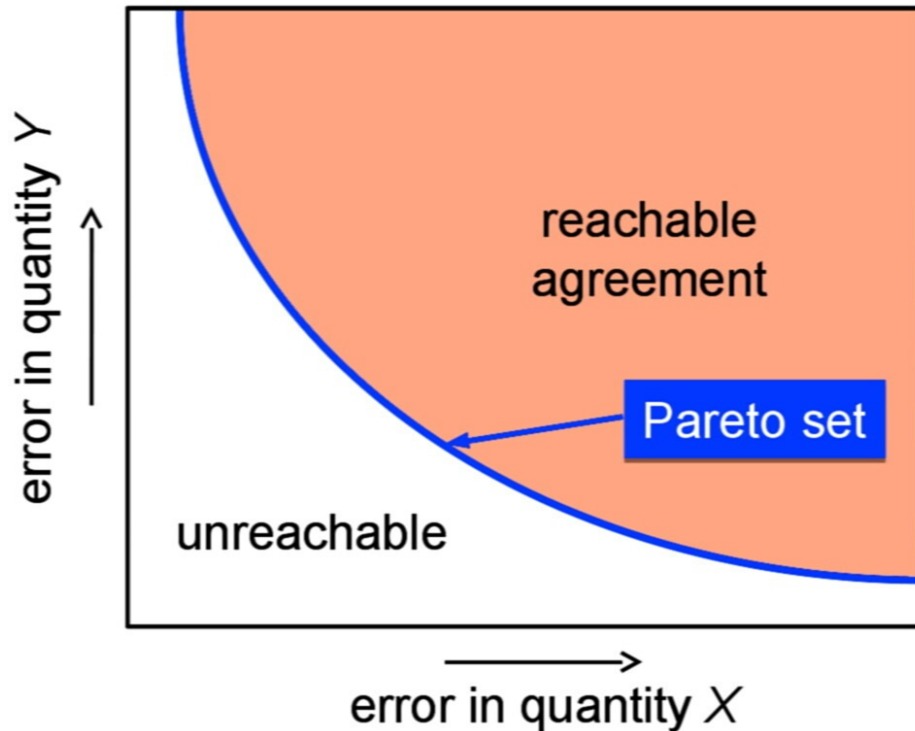
ethylene oxide model by Eckl *et al.* (2008)



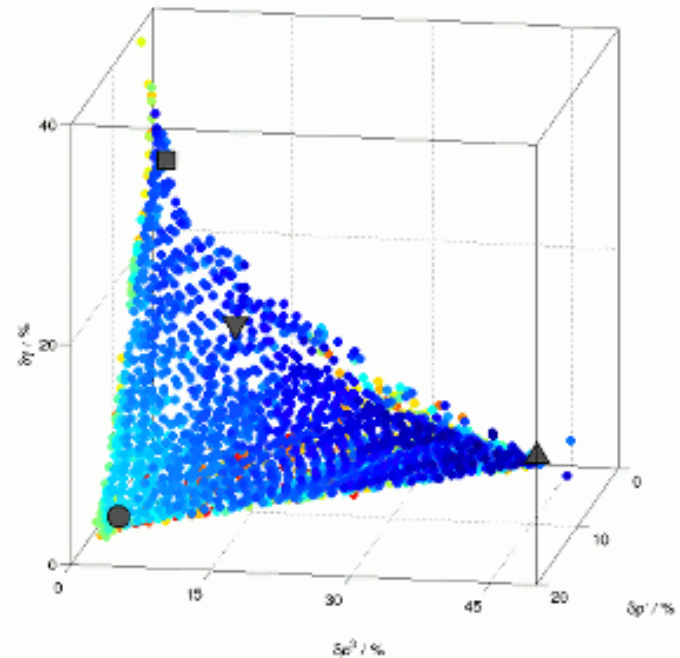


# Multicriteria optimization

Pareto optimality criterion



Multiple objectives



(2CLJQ for carbon dioxide)

Multicriteria optimization requires massively parallel molecular modelling.

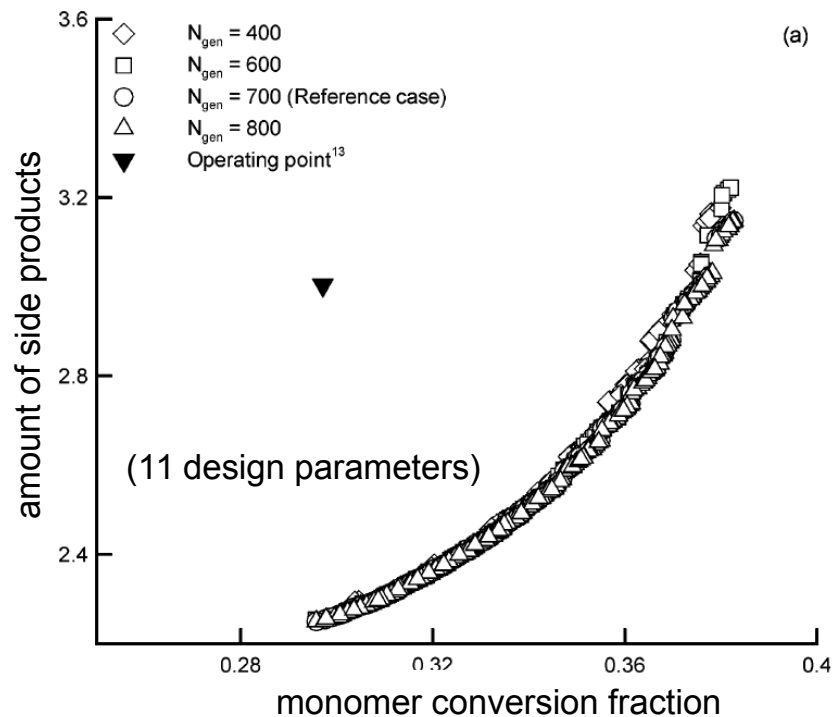




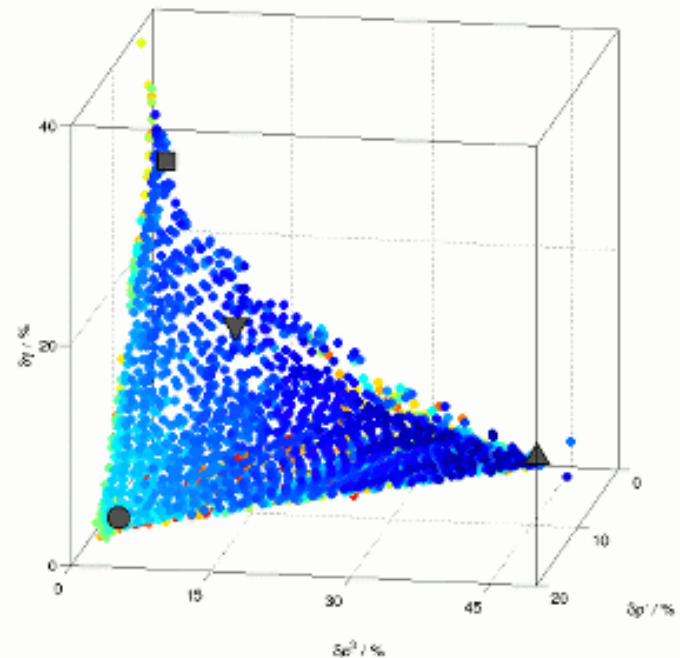
# Multicriteria optimization

## Literature example: LDPE synthesis

Ind. Eng. Chem. Res., Vol. 45, No. 9, 2006 3191



## Multiple objectives



(2CLJQ for carbon dioxide)

High-dimensional parameter spaces require stochastic exploration methods.



# Parameter space and objective space

## $p$ model parameters

(here,  $p = 4$ )

- LJ size parameter  $\sigma$
- LJ energy parameter  $\varepsilon$
- Model elongation  $L$
- Multipole moment  $\mu$  or  $Q$

Dimension of Pareto set  $d \leq p$ .

## $q$ optimization criteria

(here,  $q = 3$ )

- Saturated liquid density  $\rho'$
- Saturated vapour pressure  $p^s$
- Vapour-liquid surface tension  $\gamma$

Dimension of the Pareto set cannot be greater than  $q - 1$ .

**In general,  $d = \min(p, q - 1)$ .**

(here,  $d = 2$ )



# Computation of the Pareto set

## Multicriteria optimization problem

Simultaneously minimized objective functions  $f_\xi$  with  $\xi \in \{\rho', \rho^s, \gamma\}$  given by

$$f_\xi = \langle \delta \xi^2 \rangle_{0.55T_c^{\text{exp}} < T < 0.95T_c^{\text{exp}}} = \lim_{N \rightarrow \infty} \frac{1}{N+1} \sum_{i=0}^N \left( 1 - \frac{\xi^{\text{sim}}(T)}{\xi^{\text{exp}}(T)} \right)^2_{T/T_c = 0.55 + 0.4i/N} \quad (\text{here: } N = 9).$$

## Sandwiching

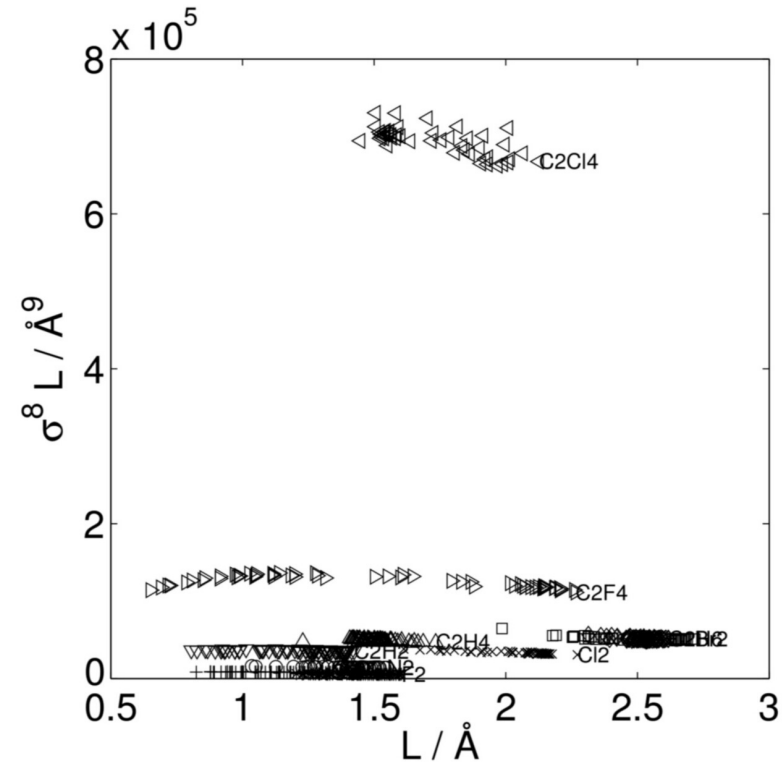
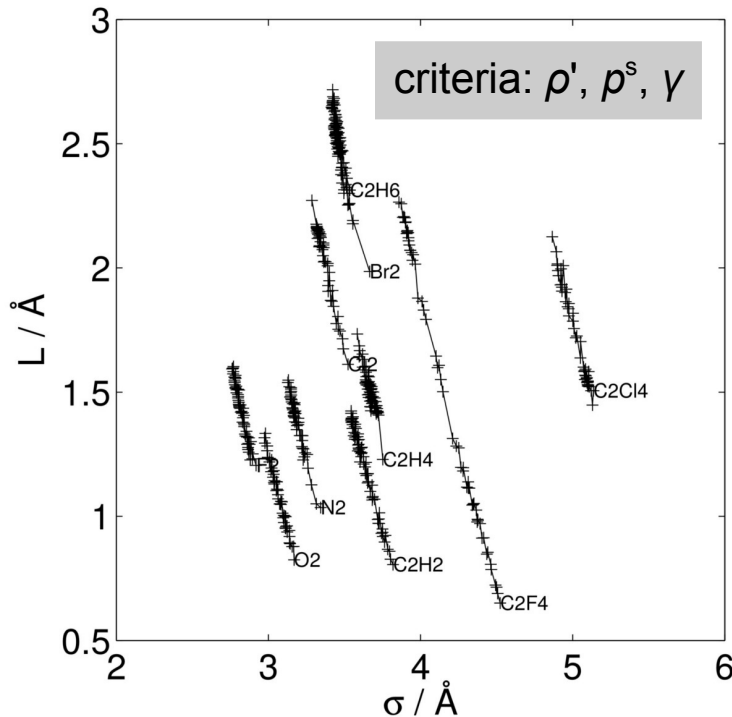
Alternating construction of inner (reachable) and outer (unreachable) approximations, assuming *local convexity* of the Pareto set.

## Hyperboxing

In non-convex regions (“hyperboxes”), Pascoletti-Serafini scalarization is employed to obtain a suitable local single-criterion optimization problem.



# Invariants of Pareto-optimal models

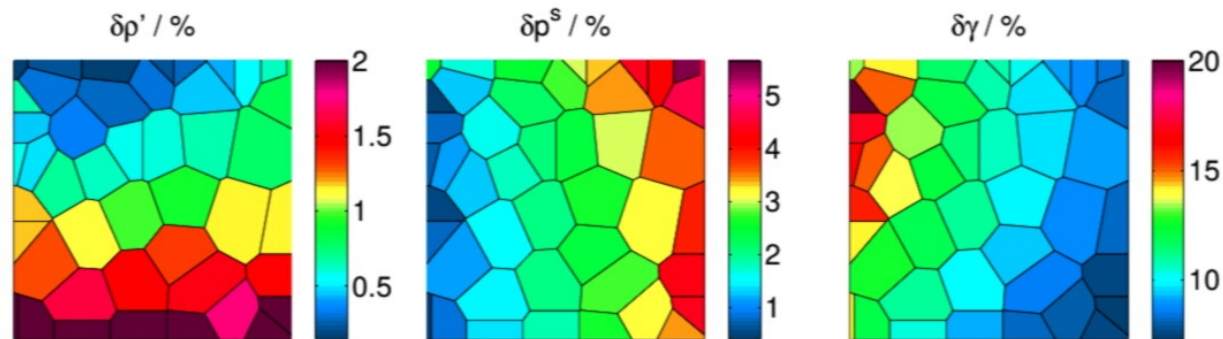


For obtaining a rough approximation of the Pareto set, the dimension of the parameter space can be reduced from four to three (or even two).

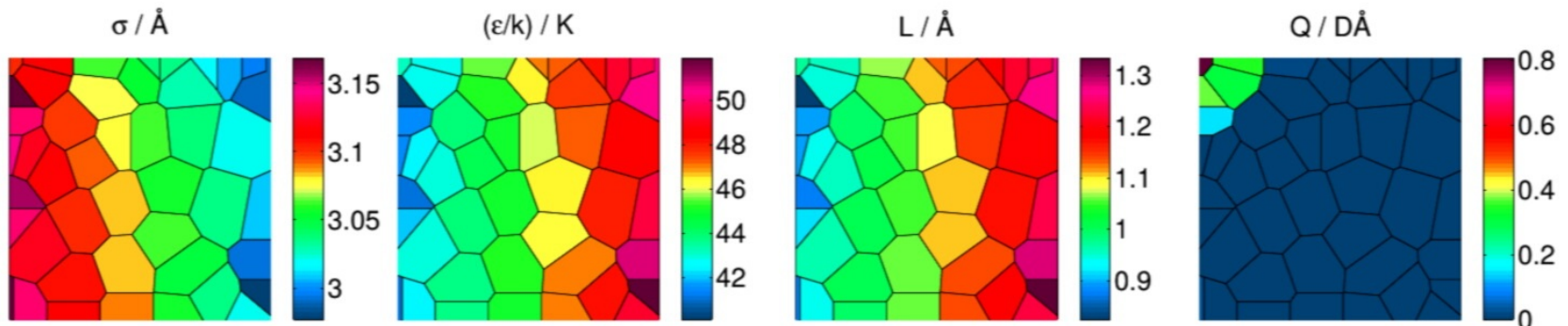


# Model tailoring by the end user

Representation of objective and parameter spaces by **patch plots**:



Pareto-optimal 2CLJQ models of molecular oxygen

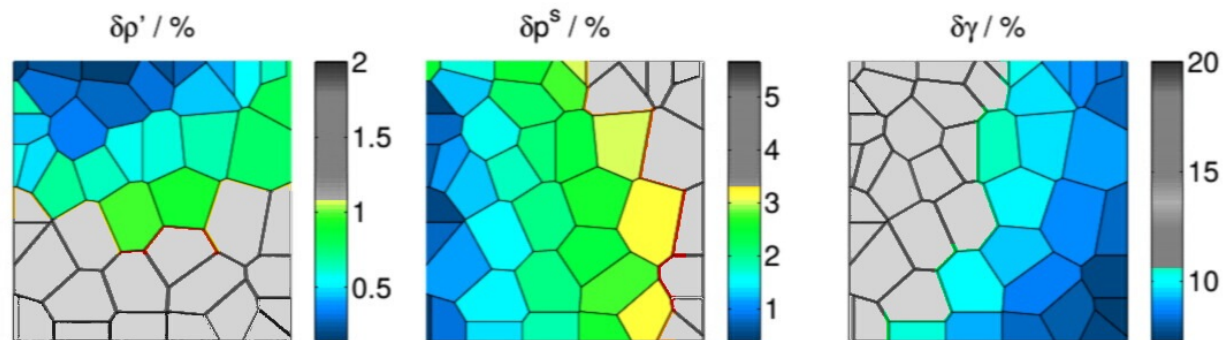




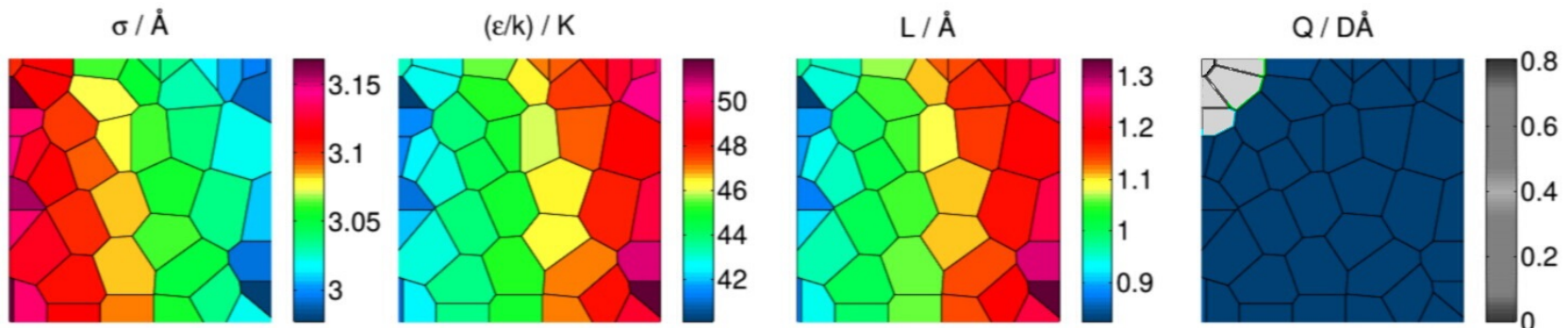


# Model tailoring by the end user

For each specific application, accuracy requirements can be specified:



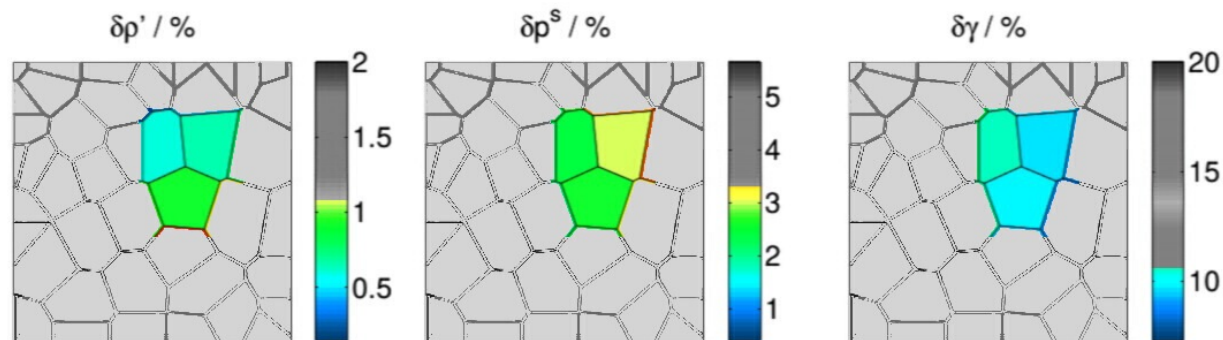
Restrictions imposed on 2CLJ models of molecular oxygen



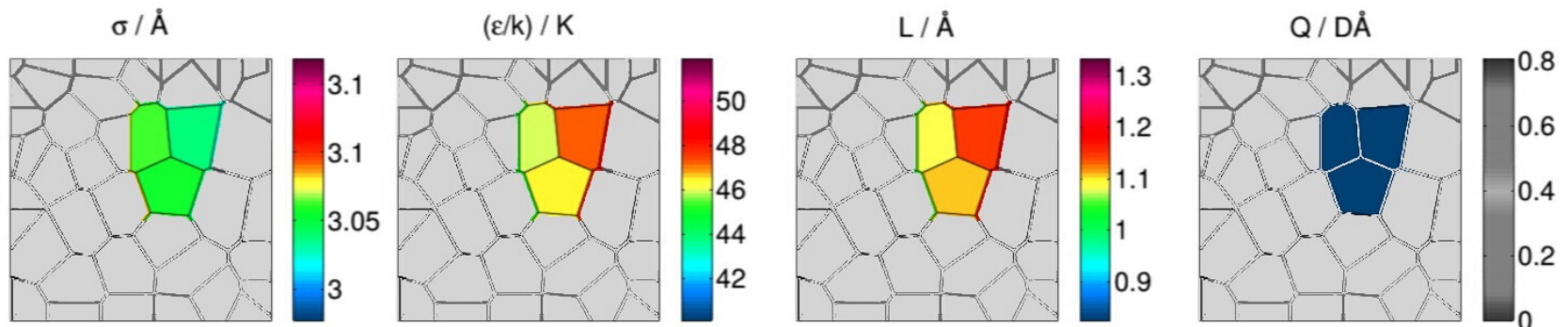


# Model tailoring by the end user

Intersection of the highlighted areas within all replicas of the patch plot:

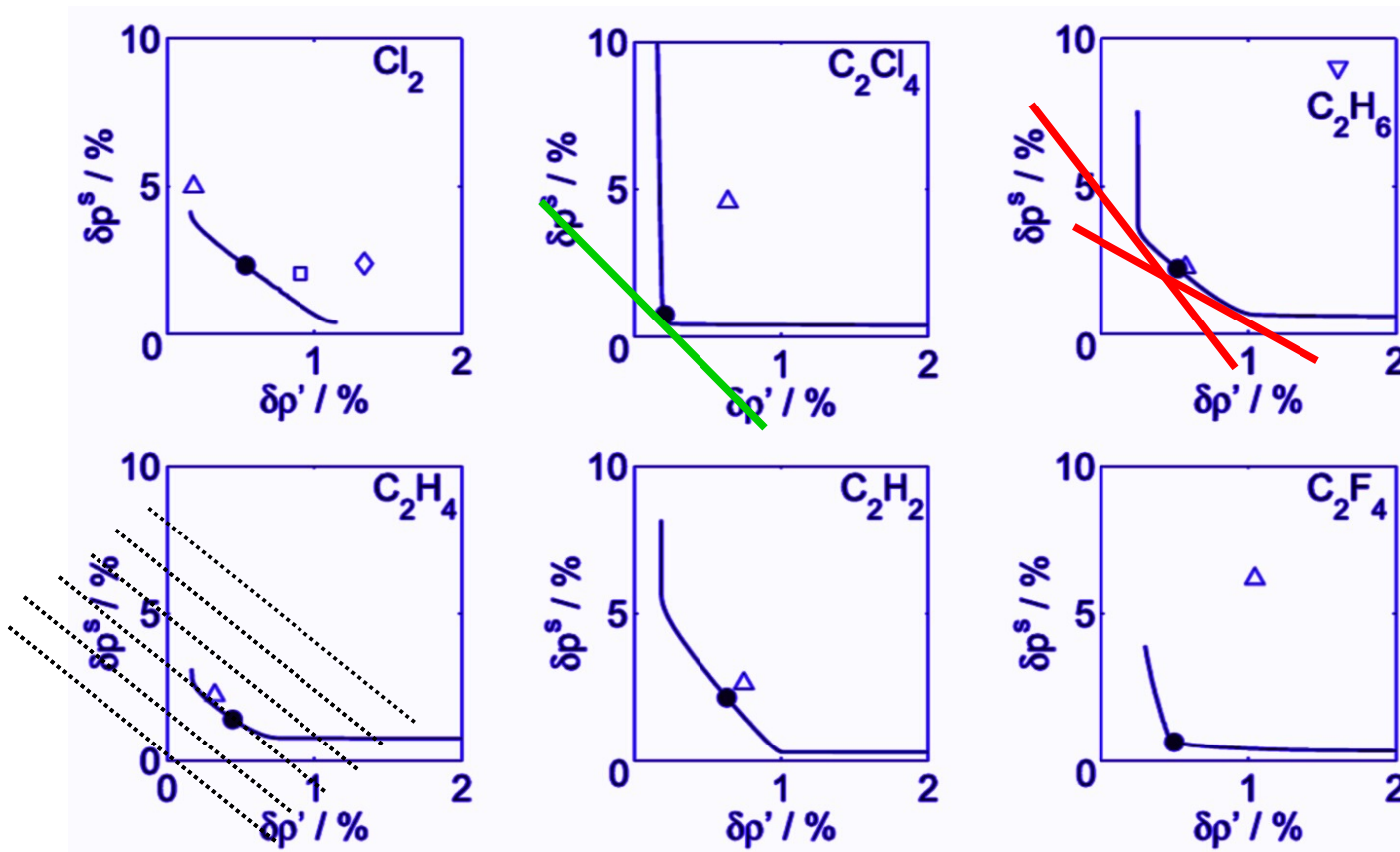


2CLJ models of molecular oxygen fulfilling all requirements





# Overall compromise models: Pareto knee



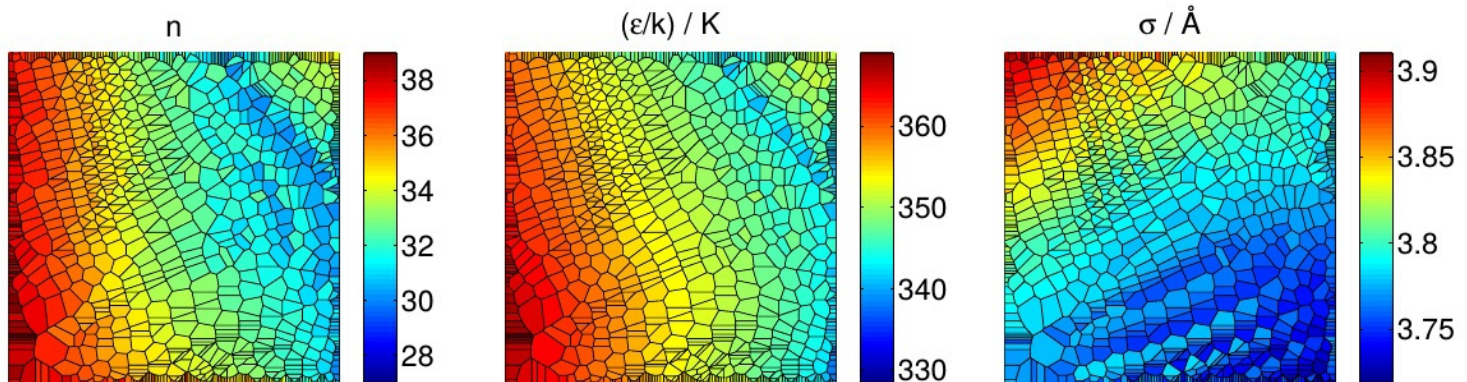
A single-criterion optimization algorithm would often miss the Pareto knee.



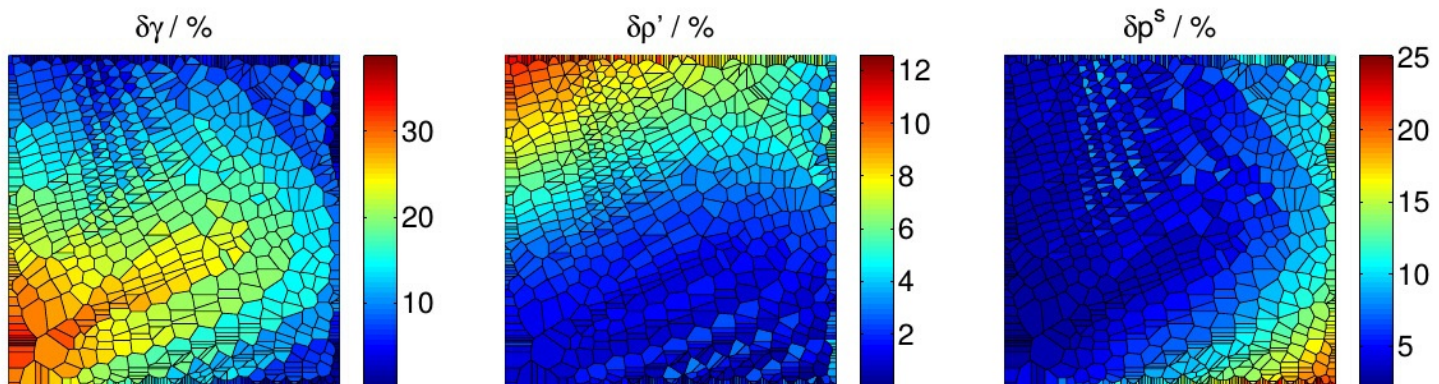
# Comparison between model classes

Carbon dioxide: Mie- $n,6$  potential 
$$u(r) = \frac{n}{n-6} \left( \frac{n}{6} \right)^{\frac{6}{n-6}} \epsilon \left[ \left( \frac{\sigma}{r} \right)^n - \left( \frac{\sigma}{r} \right)^6 \right]$$

parameter  
space

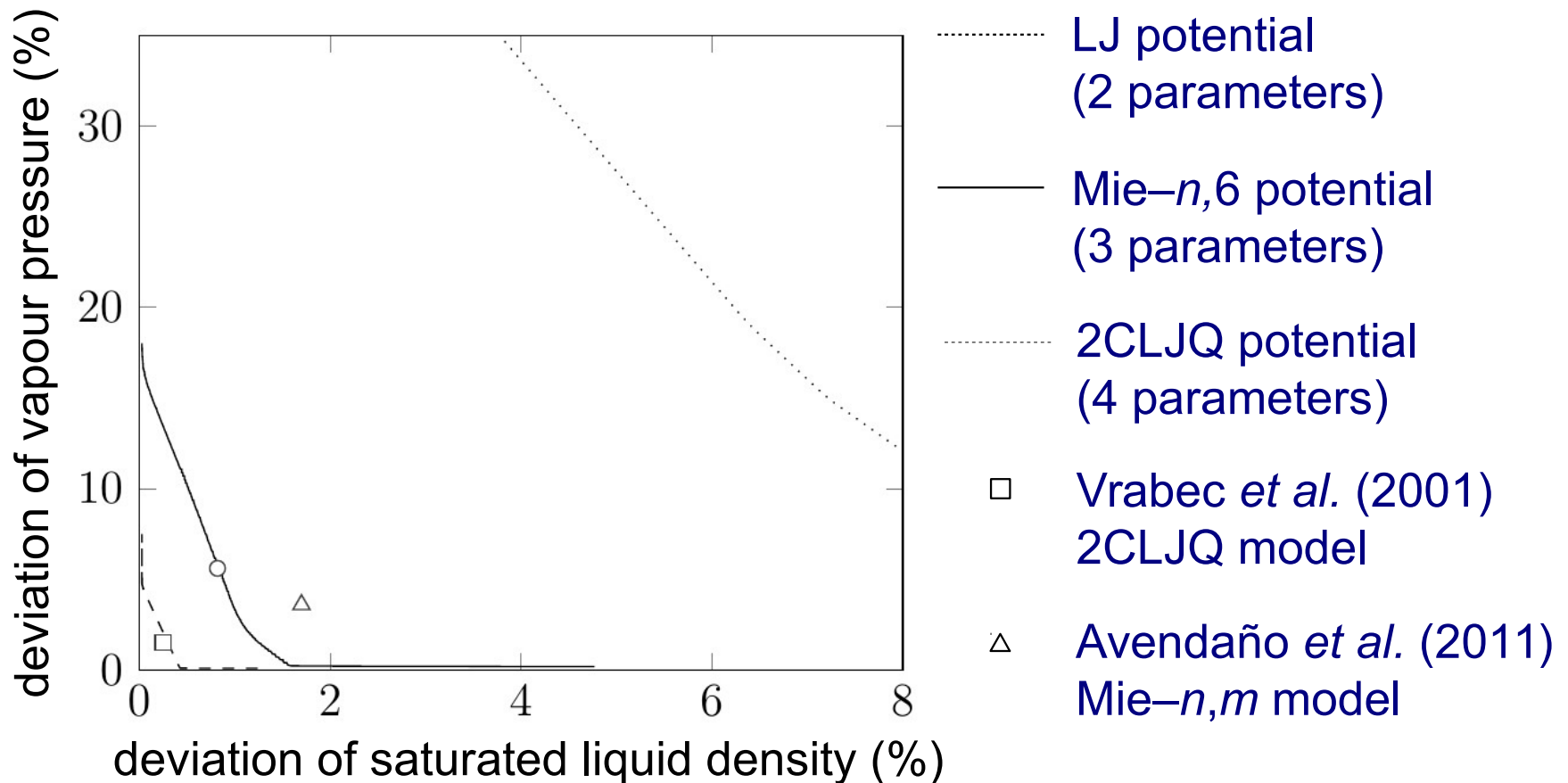


objective  
space



# Comparison between model classes

## Carbon dioxide: Mie- $n,6$ potential ./. other model classes







# Summary

## The traditional art of molecular modelling

An **expert modelling artist** designs and publishes

- a single optimized model for a particular fluid,
- according to his choice of criteria (often unknown to the public),
- users are passive, they have to live with the artists' decision.



# Summary

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## Scientific modelling by multicriteria optimization

For established model classes and multiple thermodynamic criteria,

- the dependence of thermodynamic properties on the model parameters is determined and correlated,
- the deviation between model properties and real fluid behaviour is characterized, and the Pareto set is published,
- users can design their own tailored model **with minimal effort.**