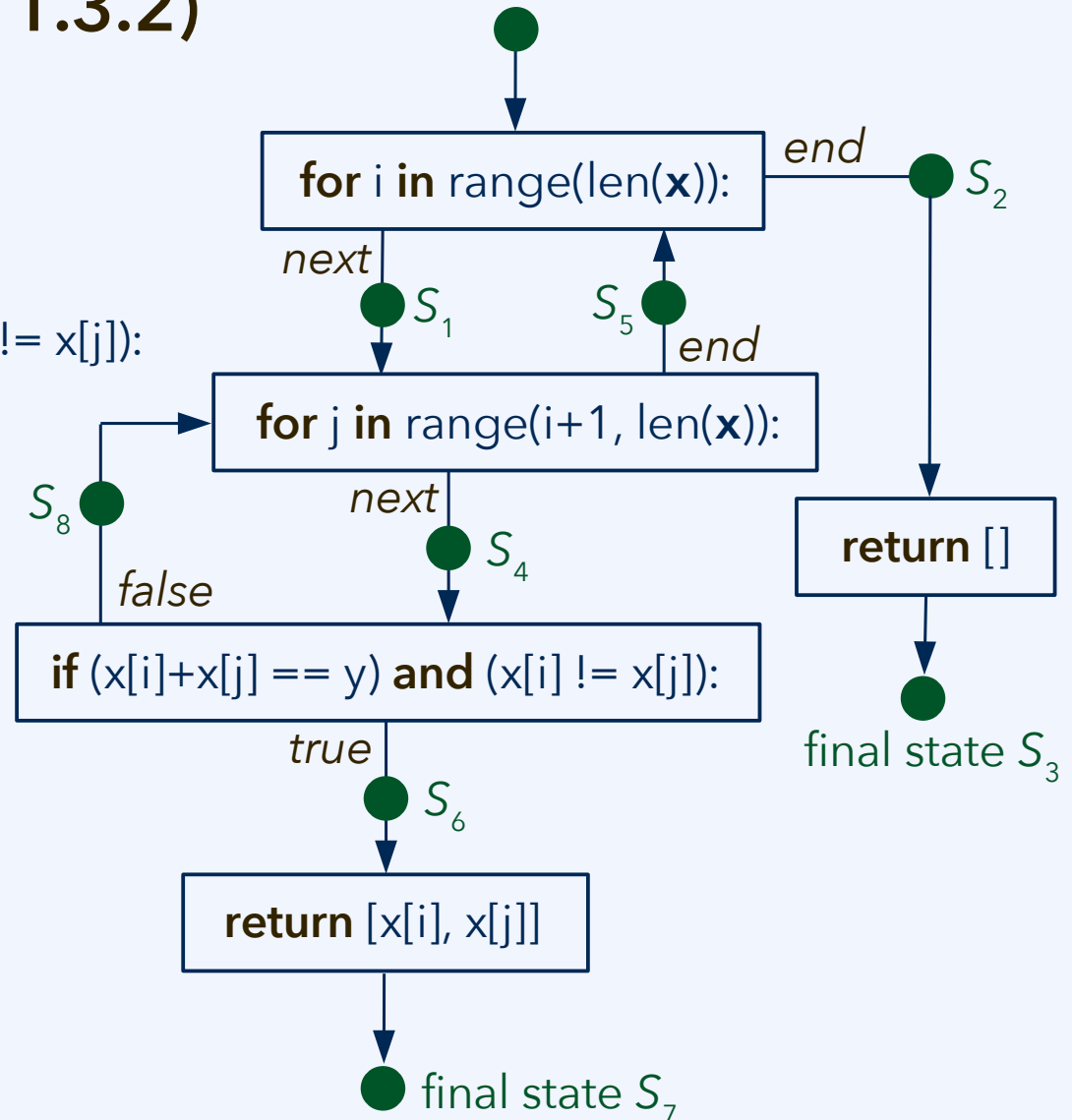


Number matching (T1.3.2)

```
def natmatch(x, y):
    for i in range(len(x)):
        for j in range(i+1, len(x)):
            if (x[i]+x[j] == y) and (x[i] != x[j]):
                return [x[i], x[j]]
    return []
```

Specification

The function takes a list \mathbf{x} and a natural number y as arguments. If in the list \mathbf{x} , there are elements a and b which are not equal and add up to y , the list $[a, b]$ is returned; otherwise, $[\]$ is returned.



Number matching (T1.3.2)

```
def natmatch(x, y):
```

```
    for i in range(len(x)):
```

```
        for j in range(i+1, len(x)):
```

```
            if (x[i]+x[j] == y) and (x[i] != x[j]):
```

```
                return [x[i], x[j]]
```

```
    return []
```

Note: Input size n given by $\text{len}(x)$

loop executed $O(n)$ times:

– loop executed $O(n)$ times:

- $O(1)$ instructions
- $O(1)$ optional instructions

$O(1)$ optional instructions

$O(n) \cdot O(n-1) + O(1) = O(n^2)$ instructions

$O(n^2)$ time efficiency

Number matching (T1.3.2)

Improved algorithm implemented by Harry Rowan:

```
def natmatch(x, y):  
    mydict = {}  
    for i in range(len(x)):  
        c = y - x[i]  
        if c in mydict:  
            return [c, x[i]]  
        mydict[x[i]] = i  
    return []
```

Python dictionaries and sets could be used to this effect equivalently.

Example, $x = [6, 4, 5, 3, 9]$, $y = 11$:

- 6 → $11 - 6 = 5$ not found in storage
insert 6 into storage
- 4 → $11 - 4 = 7$ not found in storage
insert 4 into storage
- 5 → $11 - 5 = 6$ found in storage
return [6, 5]

Number matching (T1.3.2)

Improved algorithm implemented by Harry Rowan:

```
def natmatch(x, y):
    mydict = {}
    for i in range(len(x)):
        c = y - x[i]
        if c in mydict:
            return [c, x[i]]
        mydict[x[i]] = i
    return []
```

Python dictionaries and sets are implemented as dynamically resized **hash tables**:

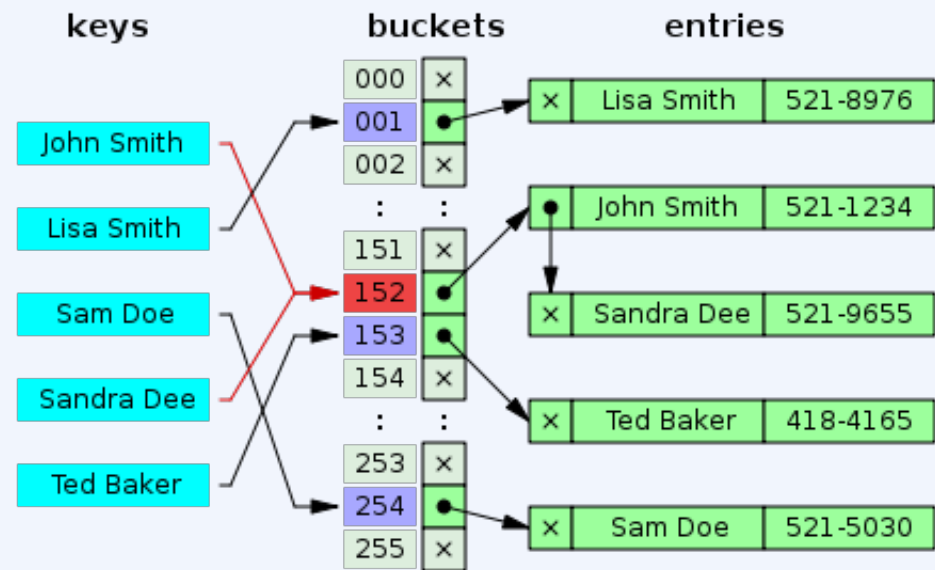


Fig. from Wikipedia, "Hash table"

Number matching (T1.3.2)

Improved algorithm implemented by Harry Rowan; worst case still $O(n^2)$:

```
def natmatch(x, y):
    mydict = {}
    for i in range(len(x)):
        c = y - x[i]
        if c in mydict:
            return [c, x[i]]
        mydict[x[i]] = i
    return []
```

In the worst case, this data structure has $O(n)$ time for search and insertion. For the average case, it is highly efficient.

Python dictionaries and sets are implemented as dynamically resized **hash tables**:

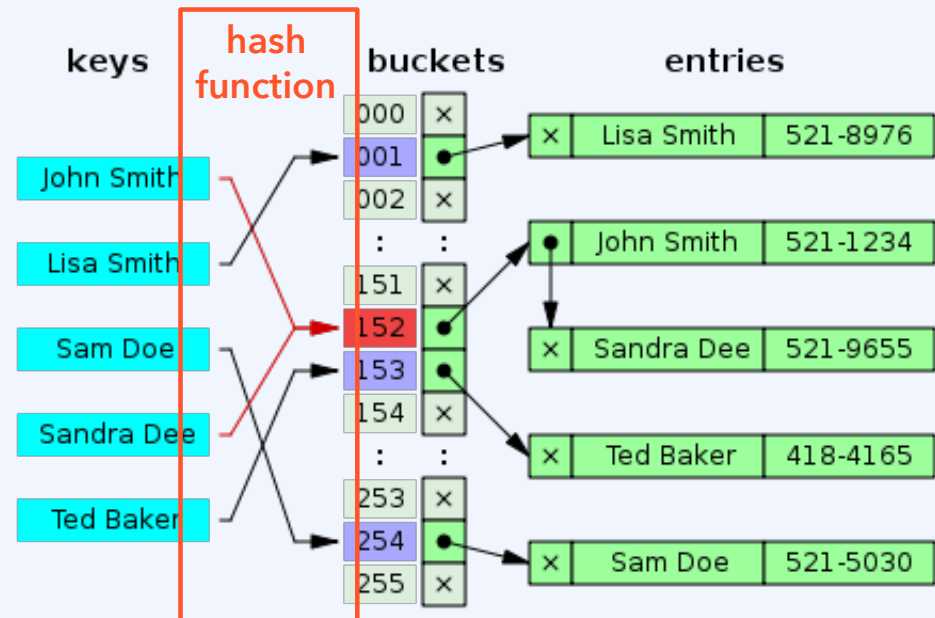


Fig. from Wikipedia, "Hash table"

Number matching (T1.3.2)

Improved algorithm implemented by Harry Rowan:

```
def natmatch(x, y):  
    initialize empty storage  
    for i in range(len(x)):  
        c = y - x[i]  
        if storage.contains(c):  
            return [c, x[i]]  
        storage.insert(x[i])  
    return []
```

$O(n)$ loop operations.

Each with one **search** operation
and one **insertion** operation.

What is the time efficiency? How does it depend on the employed data structure?

Example, $x = [6, 4, 5, 3, 9]$, $y = 11$:

6 → $11 - 6 = 5$ not found in storage
insert 6 into storage

4 → $11 - 4 = 7$ not found in storage
insert 4 into storage

5 → $11 - 5 = 6$ found in storage
return [6, 5]

Number matching (T1.3.2)

Improved algorithm implemented by Harry Rowan:

```
def natmatch(x, y):  
    initialize empty storage  
    for i in range(len(x)):  
        c = y - x[i]  
        if storage.contains(c):  
            return [c, x[i]]  
        storage.insert(x[i])  
    return []
```

$O(n)$ loop operations.

Each with one **search** operation
and one **insertion** operation.

What is the time efficiency? How does it depend on the employed data structure?

How about:

- Unsorted linked list or dyn. array?
- A sorted dynamic array?
- A sorted linked list or an unbalanced search tree?
- A balanced search tree?
- Python sets or dicts?

Number matching (T1.3.2)

Improved algorithm implemented by Harry Rowan:

```
def natmatch(x, y):  
    initialize empty storage  
    for i in range(len(x)):  
        c = y - x[i]  
        if storage.contains(c):  
            return [c, x[i]]  
        storage.insert(x[i])  
    return []
```

$O(n)$ loop operations.

Each with one **search (s)** operation
and one **insertion (i)** operation.

What is the time efficiency? How does it depend on the employed data structure?

How about:

- Unsorted linked list or dyn. array?
s done in $O(n)$, **i** done in $O(1)$.
- A sorted dynamic array?
s done in $O(\log n)$, **i** done in $O(n)$.
- A sorted linked list or an unbalanced search tree?
s done in $O(n)$, **i** done in $O(n)$.
- A balanced search tree?
s and **i** both done in $O(\log n)$.
- Python sets or dicts?
Worst case $O(n)$ for both **s** and **i**.

Number matching (T1.3.2)

Improved algorithm implemented by Harry Rowan:

```
def natmatch(x, y):  
    initialize empty storage  
    for i in range(len(x)):  
        c = y - x[i]  
        if storage.contains(c):  
            return [c, x[i]]  
        storage.insert(x[i])  
    return []
```

$O(n)$ loop operations.

- $O(\log n)$ time per iteration.

$O(n \log n)$ with a balanced tree.

What is the time efficiency? How does it depend on the employed data structure?

How about:

- Unsorted linked list or dyn. array?
s done in $O(n)$, **i** done in $O(1)$.
- A sorted dynamic array?
s done in $O(\log n)$, **i** done in $O(n)$.
- A sorted linked list or an unbalanced search tree?
s done in $O(n)$, **i** done in $O(n)$.
- A balanced search tree?
s and **i** both done in $O(\log n)$.
- Python sets or dicts?
Worst case $O(n)$ for both **s** and **i**.