University of Central Lancashire
UCLan

Resolution: Tutorial 4.5 problem

CO2412

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4.5.1 Concepts

Literals: Atomic statements p, q, …, and their negations $\neg p$, $\neg q$, …

Clauses: A conjunction ("and") of literals, such as $p \land \neg q \land \neg r$, is a conjunctive clause. A disjunction ("or") of literals, such as $\neg p \lor q \lor r$, is a disjunctive clause.

Conjunctive normal form (CNF):

- A statement is in CNF if it is a **conjunction of disjunctive clauses**.
- It is in **full CNF** if all atomic statements appear in all disjunctive clauses.
- The full CNF version of a truth table has one clause per **False** valuation.

Entailment: R entails S if and only if every model of R is a model of S. ($R \vDash S$.)

Inference: Deduction of an entailment following a rule or a system of rules.

Resolution: Inference technique applied to CNF statements based on the rule

$$(p \lor L_0 \lor L_1 \lor \ldots) \land (\neg p \lor M_0 \lor M_1 \lor \ldots) \vDash (L_0 \lor L_1 \lor \ldots \lor M_0 \lor M_1 \lor \ldots).$$

4.5.1 Resolution (completeness for satisfiability)

Completeness of resolution:

- If a statement in CNF is a **contradiction**, an algorithm implementing resolution as an inference method **succeeds at proving this in all cases;** *i.e.*, two clauses p_i and $\neg p_i$ for the same atomic statement are deduced.
- The same applies to proving that multiple statements are **inconsistent**.
- If resolution does not detect a contradiction, the statement is **satisfiable**.
- To check whether R is a **tautology**, resolution can be applied to $\neg R$.

Entailment: R entails S if and only if every model of R is a model of S. ($R \vDash S$.)

Inference: Deduction of an entailment following a rule or a system of rules.

Resolution: Inference technique applied to CNF statements based on the rule

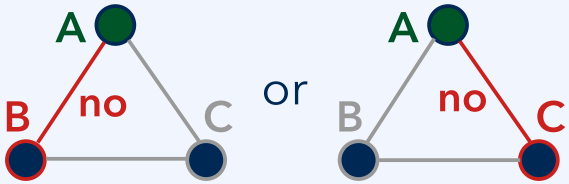
$(p \lor L_0 \lor L_1 \lor \ldots) \land (\neg p \lor M_0 \lor M_1 \lor \ldots) \vDash (L_0 \lor L_1 \lor \ldots \lor M_0 \lor M_1 \lor \ldots)$.

4.5.2 From logic to graphs

Undirected graph; p_{AB} representing “there is an edge between A and B,” *etc.*

How would we paraphrase the meaning of the propositional logic statements:

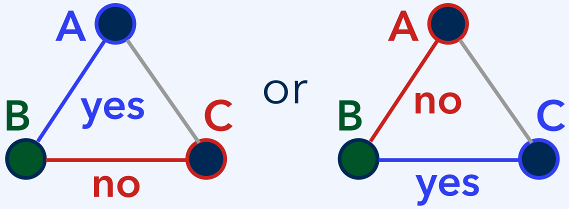
$S_A = \neg p_{AB} \vee \neg p_{AC}$?



“Vertex A does not have degree 2.”

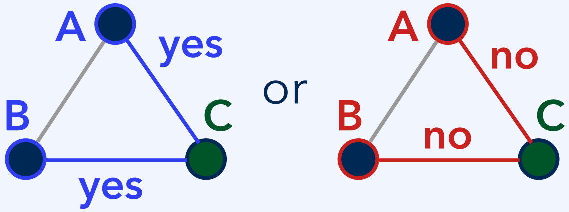
“Vertex A has degree 0 or 1.”

$S_B = p_{AB} \leftrightarrow \neg p_{BC}$?



“Vertex B has degree 1.”

$S_C = p_{AC} \leftrightarrow p_{BC}$?



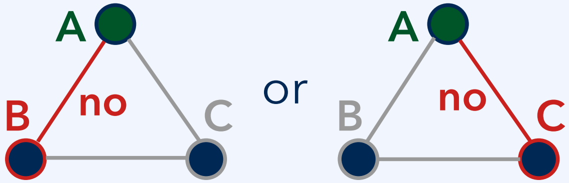
“Vertex C has degree 0 or 2.”

How many literals are there? Six: p_{AB}, $\neg p_{AB}$, p_{AC}, $\neg p_{AC}$, p_{BC}, and $\neg p_{BC}$.

4.5.3 Conjunctive normal form

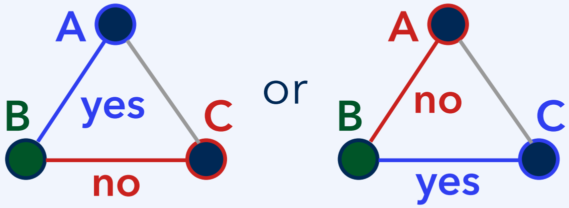
Transformation to CNF. Rule: $(R \leftrightarrow S) \equiv (R \lor \neg S) \land (\neg R \lor S)$.

$S_A = \neg p_{AB} \lor \neg p_{AC}$



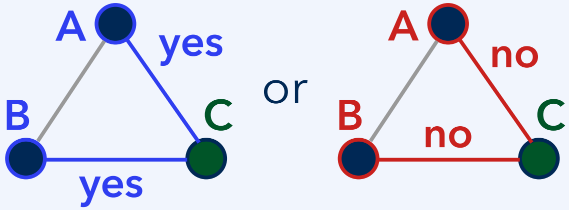
(already in CNF)

$S_B = p_{AB} \leftrightarrow \neg p_{BC}$



$$p_{AB} \leftrightarrow \neg p_{BC}$$
$$\equiv (p_{AB} \lor \neg\neg p_{BC}) \land (\neg p_{AB} \lor \neg p_{BC})$$
$$\equiv (p_{AB} \lor p_{BC}) \land (\neg p_{AB} \lor \neg p_{BC})$$

$S_C = p_{AC} \leftrightarrow p_{BC}$



$$p_{AC} \leftrightarrow \neg p_{BC}$$
$$\equiv (p_{AC} \lor \neg p_{BC}) \land (\neg p_{AC} \lor p_{BC})$$

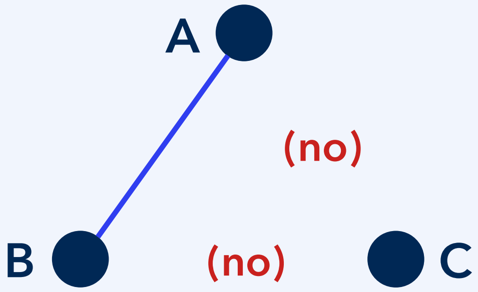
Clauses: 0) $\neg p_{AB} \lor \neg p_{AC}$ 1) $p_{AB} \lor p_{BC}$ 2) $\neg p_{AB} \lor \neg p_{BC}$ 3) $p_{AC} \lor \neg p_{BC}$ 4) $\neg p_{AC} \lor p_{BC}$

4.5.4 Consistency: Common model

If multiple statements are consistent, they have a common model.

$S_A = \neg p_{AB} \lor \neg p_{AC}$

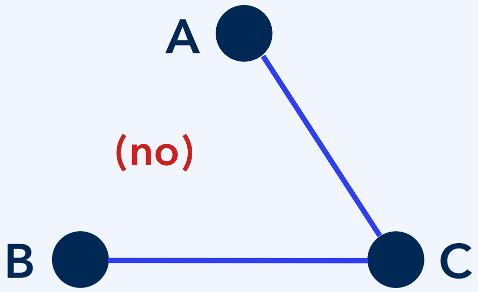
$(\neg p_{AB} \lor \neg p_{AC})$



"Vertex A has degree 0 or 1."

$S_B = p_{AB} \leftrightarrow \neg p_{BC}$

$(p_{AB} \lor p_{BC}) \land (\neg p_{AB} \lor \neg p_{BC})$



"Vertex B has degree 1."

$S_C = p_{AC} \leftrightarrow p_{BC}$

$(p_{AC} \lor \neg p_{BC}) \land (\neg p_{AC} \lor p_{BC})$

"Vertex C has degree 0 or 2."

Clauses: 0) $\neg p_{AB} \lor \neg p_{AC}$ 1) $p_{AB} \lor p_{BC}$ 2) $\neg p_{AB} \lor \neg p_{BC}$ 3) $p_{AC} \lor \neg p_{BC}$ 4) $\neg p_{AC} \lor p_{BC}$

4.5.4 Consistency: Resolution

$$\boxed{(p \lor L_0 \lor L_1 \lor \ldots) \land (\neg p \lor M_0 \lor M_1 \lor \ldots) \models (L_0 \lor L_1 \lor \ldots \lor M_0 \lor M_1 \lor \ldots).}$$

0) $\neg p_{AB} \lor \neg p_{AC}$ with 1) $p_{AB} \lor p_{BC}$ | resolves to 4) $\neg p_{AC} \lor p_{BC}$

1) $p_{AB} \lor p_{BC}$ with 2) $\neg p_{AB} \lor \neg p_{BC}$ | resolves to $p_{AB} \lor \neg p_{AB}$ and $p_{BC} \lor \neg p_{BC}$

```
i = 1
while i < len(clauses):
    for j in range(i):
        if direct_contradiction(clauses[i], clauses[j]):
            return False
        resolved_clauses = resolve(clauses[i], clauses[j])
        for c in resolved_clauses:
            if should_be_appended(c, clauses):
                clauses.append(c)
    i += 1
return True
```

contradiction found if the two clauses are single opposite literals, such as p_2 and $\neg p_2$

append if the resolved clause is not redundant and not tautological

Clauses: 0) $\neg p_{AB} \lor \neg p_{AC}$ 1) $p_{AB} \lor p_{BC}$ 2) $\neg p_{AB} \lor \neg p_{BC}$ 3) $p_{AC} \lor \neg p_{BC}$ 4) $\neg p_{AC} \lor p_{BC}$

4.5.4 Consistency: Resolution

$$\boxed{(p \lor L_0 \lor L_1 \lor \dots) \land (\neg p \lor M_0 \lor M_1 \lor \dots) \vDash (L_0 \lor L_1 \lor \dots \lor M_0 \lor M_1 \lor \dots).}$$

0) $\neg p_{AB} \lor \neg p_{AC}$ with 1) $p_{AB} \lor p_{BC}$ resolves to 4) $\neg p_{AC} \lor p_{BC}$

1) $p_{AB} \lor p_{BC}$ with 2) $\neg p_{AB} \lor \neg p_{BC}$ resolves to $p_{AB} \lor \neg p_{AB}$ and $p_{BC} \lor \neg p_{BC}$

0) $\neg p_{AB} \lor \neg p_{AC}$ with 3) $p_{AC} \lor \neg p_{BC}$ resolves to 2) $\neg p_{AB} \lor \neg p_{BC}$

1) $\boldsymbol{p_{AB} \lor p_{BC}}$ with 3) $\boldsymbol{p_{AC} \lor \neg p_{BC}}$ resolves to 5) $\boldsymbol{p_{AB} \lor p_{AC}}$

2) $\neg p_{AB} \lor \neg p_{BC}$ with 4) $\neg p_{AC} \lor p_{BC}$ resolves to 0) $\neg p_{AB} \lor \neg p_{AC}$

3) $p_{AC} \lor \neg p_{BC}$ with 4) $\neg p_{AC} \lor p_{BC}$ resolves to $p_{AC} \lor \neg p_{AC}$ and $p_{BC} \lor \neg p_{BC}$

0) $\neg p_{AB} \lor \neg p_{AC}$ with 5) $p_{AB} \lor p_{AC}$ resolves to $p_{AB} \lor \neg p_{AB}$ and $p_{AC} \lor \neg p_{AC}$

2) $\neg p_{AB} \lor \neg p_{BC}$ with 5) $p_{AB} \lor p_{AC}$ resolves to 3) $p_{AC} \lor \neg p_{BC}$

4) $\neg p_{AC} \lor p_{BC}$ with 5) $p_{AB} \lor p_{AC}$ resolves to 1) $p_{AB} \lor p_{BC}$

Clauses: 0) $\neg p_{AB} \lor \neg p_{AC}$ 1) $p_{AB} \lor p_{BC}$ 2) $\neg p_{AB} \lor \neg p_{BC}$ 3) $p_{AC} \lor \neg p_{BC}$ 4) $\neg p_{AC} \lor p_{BC}$