Example modelling problem

We have seen two visualization techniques. Using the second technique, each parameter and each objective is shown in its own heat map. Only the Pareto optimal solutions are shown; each solution corresponds to a field on the map.

The code from the **pareto-front-visualization** notebook is an ad-hoc creation. We will now go through the changes that need to be made when using it.

Example modelling scenario (based on Katib Hussain's case):

"The university has considered to upgrade workstations over the Christmas holidays (2 weeks). Up to 600 workstations would need to be upgraded, but for every ten workstations, a day's maintenance is required."

"A large maintenance team with too little investment would reduce productivity and reduce the amount of workstation upgrades. Too much investment with a small team would limit the use that can be made of the equipment."

Example modelling problem (T2.1 discussion)

We have seen two visualization techniques. Using the second technique, each parameter and each objective is shown in its own heat map. Only the Pareto optimal solutions are shown; each solution corresponds to a field on the map.

The code from the **pareto-front-visualization** notebook is an ad-hoc creation. We will now go through the changes that need to be made when using it.

Example modelling scenario (based on Katib Hussain's case):

Minimization objectives:

- y_0, expenses of an upgrade/maintenance operation for workstations
- y_1, number of workstations (out of 600) that do not receive an upgrade

"A large maintenance team with too little investment would reduce productivity and reduce the amount of workstation upgrades. Too much investment with a small team would limit the use that can be made of the equipment."

Example modelling problem (T2.1 discussion)

How to specify the parameter space: For each parameter, the range of permitted values needs to be specified; including constraints, if applicable.

Here, maybe simply $0 \leq x_0$ and $0 \leq x_1$.

Optional constraint, may be included explicitly or not: salary $\cdot\, x_1 \leq x_0$.

Example modelling scenario (based on Katib Hussain's case):

Minimization objectives:

- y_0, expenses of an upgrade/maintenance operation for workstations
- y_1, number of workstations (out of 600) that do not receive an upgrade

Parameters:

- x_0, expenses of an upgrade/maintenance operation for workstations
- x_1, number of staff assigned to carry out the upgrade (within two weeks)

Example modelling problem (T2.1 discussion)

What dimension do we expect for the relevant spaces and sets?

- The parameter space ($m = 2$) and objective space ($n = 2$) are both 2D.
- The accessible part of objective space ($q = 2$) will probably be 2D.
- Pareto front ($p = 1$) and set of Pareto-optimal solutions ($p' = 1$) are 1D.

Example modelling scenario (based on Katib Hussain's case):

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- y_0, expenses of an upgrade/maintenance operation for workstations
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Parameters:

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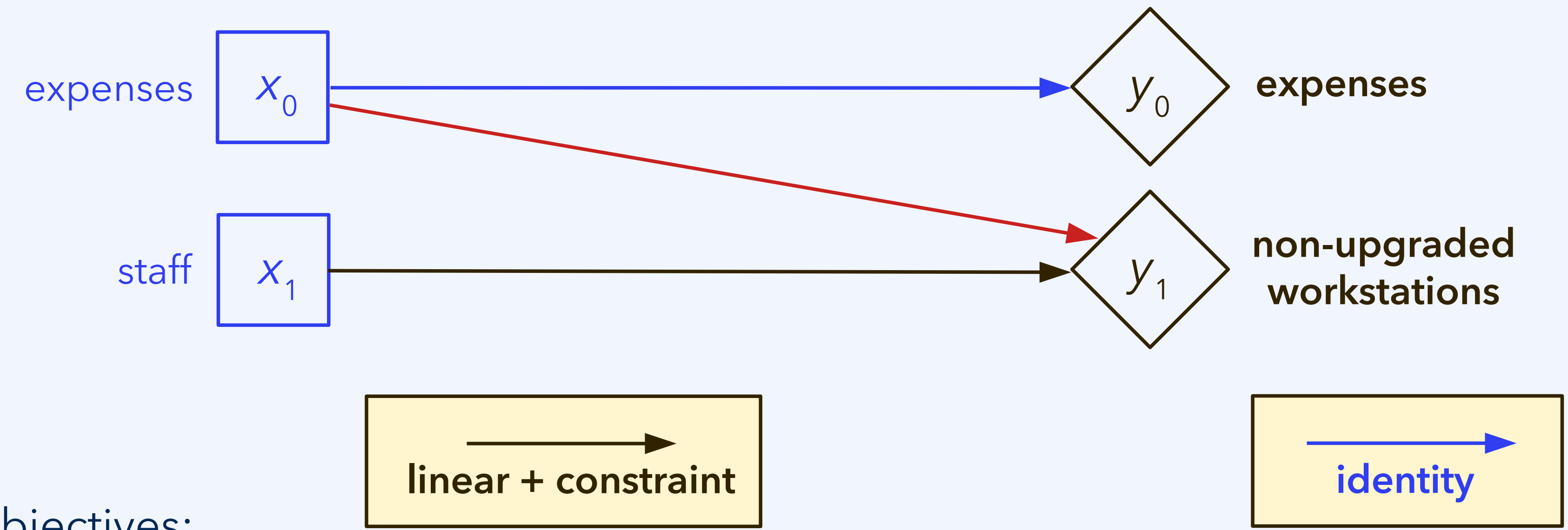
Example modelling problem (T2.1 discussion)



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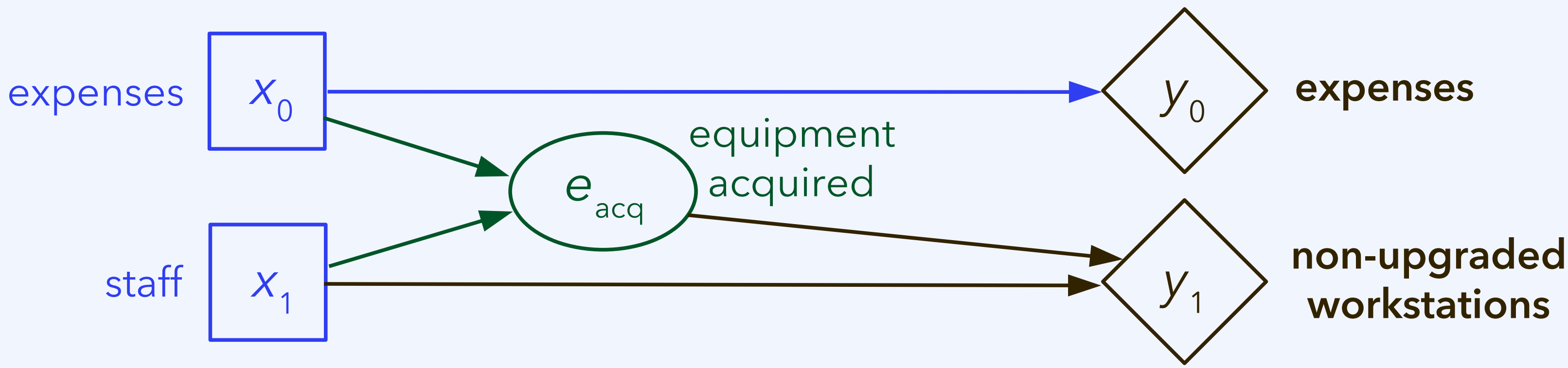
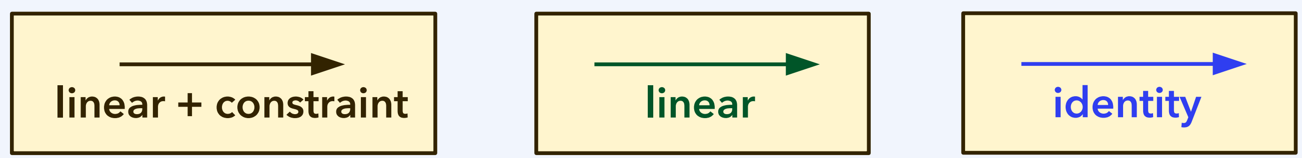
Example modelling problem (T2.1 discussion)





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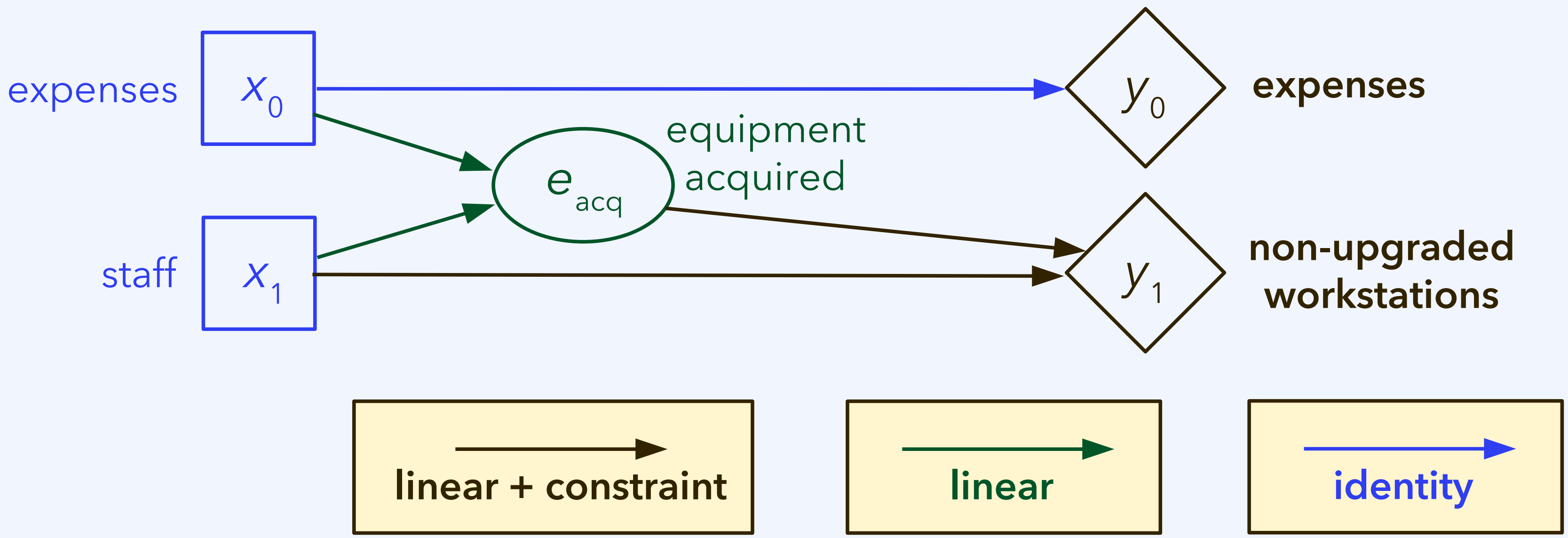
Example modelling problem (T2.1 discussion)



$y_0(x_0) = x_0$

$e_{acq}(x_0, x_1) = (x_0 - \text{salary} \cdot x_1) / \text{unit\_cost}$

$y_1(e_{acq}, x_1) = \max(0, \text{num\_units} - \min(e_{acq}, x_1/\text{fte\_per\_unit}))$

Example modelling problem (T2.1 discussion)

Reasonable estimates for the **constant coefficients** included in the model:

- num_units = 600, since 600 workstations require an upgrade.
- salary = GBP 15,000, *i.e.*, 1.0 FTE for two weeks for a technician. This is strictly speaking not just the salary, it needs to include non-wage costs.
- fte_per_unit = 0.01, since "for every ten workstations, a day's maintenance is required." Hence, 1.0 FTE can fix 100 units in ten working days.
- unit_cost = GBP 150, might be reasonable for a typical maintenance.

$y_0(x_0) = x_0$

$e_{acq}(x_0, x_1) = (x_0 - \text{salary} \cdot x_1) \,/\, \text{unit\_cost}$

$y_1(e_{acq}, x_1) = \max(0, \text{num\_units} - \min(e_{acq}, x_1/\text{fte\_per\_unit}))$

Example modelling problem (T2.1 discussion)

Can this scenario be treated by multicriteria optimization?
Verify whether the optimization criteria are in mutual disagreement:

- Optimum with respect to y_0 only, ignoring y_1:
  No expenses, therefore also no staff allocated to the work ($x_0 = x_1 = 0$).
- Optimum with respect to y_1 only, ignoring y_0: Any solution where all workstations are upgraded. There, $x_1 \geq \text{num\_units} \cdot \text{fte\_per\_unit} = 6$. Also, $x_0 \geq \text{num\_units} \cdot \text{unit\_cost} + x_1 \cdot \text{salary} \geq$ GBP 180,000.

$$y_0(x_0) = x_0$$

$$e_{acq}(x_0, x_1) = (x_0 - \text{salary} \cdot x_1) \,/\, \text{unit\_cost}$$

$$y_1(e_{acq}, x_1) = \max(0, \text{num\_units} - \min(e_{acq}, x_1/\text{fte\_per\_unit}))$$

Example modelling problem (T2.1 discussion)

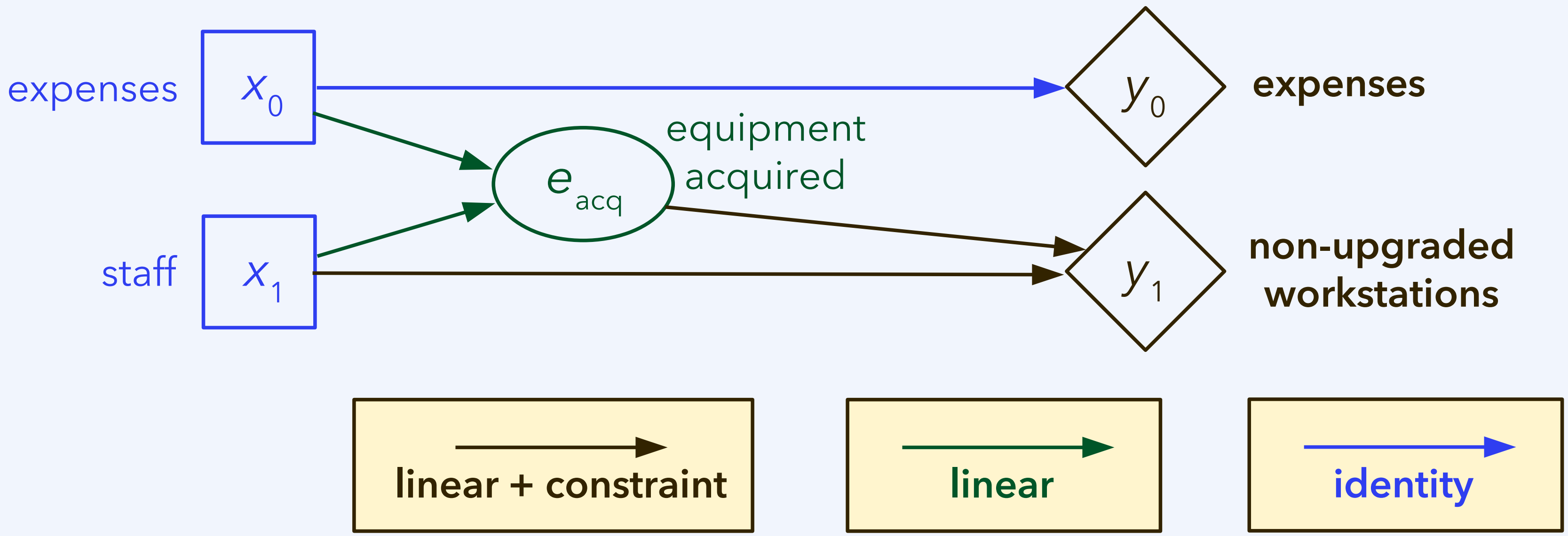
```
def cost_function(x):


    expenses = x[0]
    acquired_equipment = (x[0] - salary*x[1]) / unit_cost
    upgraded_units = min(num_units, acquired_equipment, x[1]/fte_per_unit)

    y = [expenses, num_units - upgraded_units]
    return y
```

$y_0(x_0) = x_0$

$e_{acq}(x_0, x_1) = (x_0 - \text{salary} \cdot x_1) / \text{unit\_cost}$

$y_1(e_{acq}, x_1) = \max(0, \text{num\_units} - \min(e_{acq}, x_1/\text{fte\_per\_unit}))$

Reanalysis of the problem (T2.1 discussion)

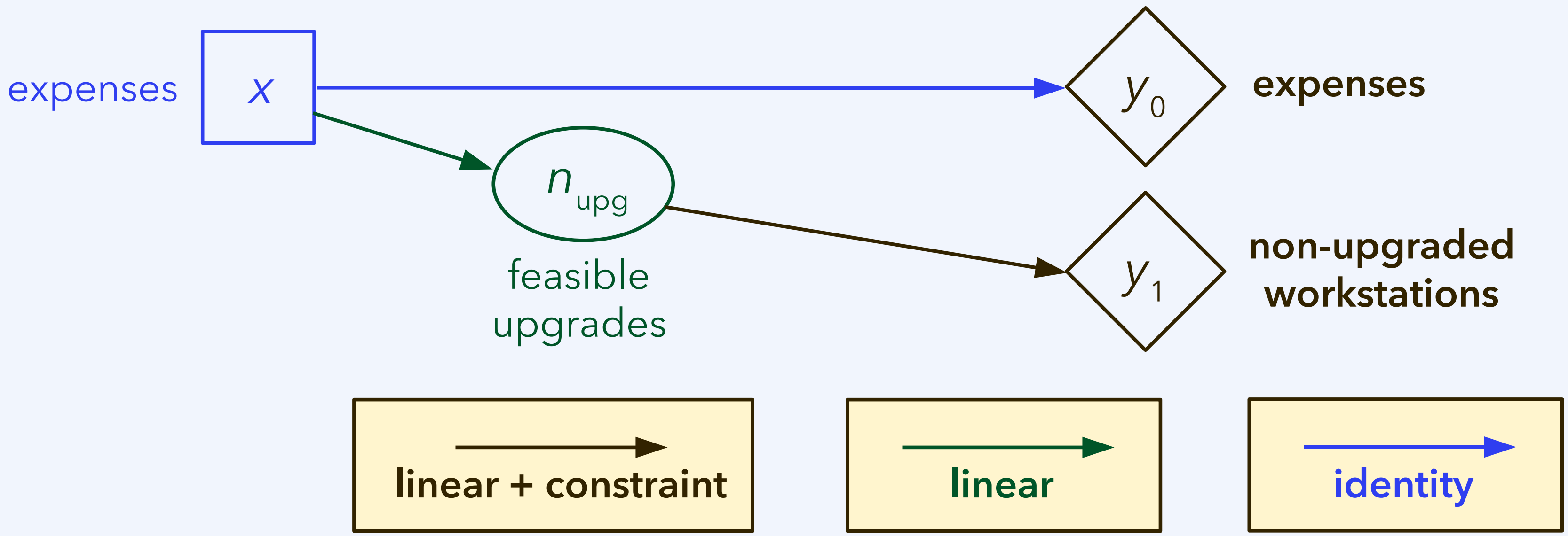


$y_0(x_0) = x_0$

$e_{acq}(x_0, x_1) = (x_0 - \text{salary} \cdot x_1) / \text{unit\_cost}$

$y_1(e_{acq}, x_1) = \max(0, \text{num\_units} - \min(e_{acq}, x_1/\text{fte\_per\_unit}))$

Reanalysis of the problem (T2.1 discussion)



$y_0(x) = x$

$n_{upg}(x) = x \,/\, (\text{unit\_cost} + \text{salary} \cdot \text{fte\_per\_unit})$

$y_1(n_{upg}) = \max(0, \text{num\_units} - n_{upg})$