## Recommendation: Solve two out of the three given problems.

- You have 110 minutes from the moment that the beginning of the exam is announced.
- This exam consists of three problems. Each is worth up to 25 credits, out of 100 credits for the whole course. At most 40 credits can be gained from the present term exam.

You need to work on two problems to achieve an optimal outcome.

- If you choose to work on two problems, the problem with the best outcome will count normally (i.e., up to 25 credits), and the other problem will be scaled by a factor of $60 \%$ (i.e., up to 15 credits), yielding an optimum total of 40 credits.
- If you choose to work on all three problems, the outcomes will be ordered by the number of credits achieved. The best problem counts with a factor of $100 \%$, and the second best problem is scaled by a factor of $60 \%$, yielding up to 40 credits for the exam as a whole. The outcome of the remaining problem does not influence your grade.
- There is also a bonus problem, yielding up to eight credits additionally.

Make sure that every paper that you submit contains your name and student ID. Any access to means of communication is a case of cheating irrespective of what is communicated. It is enough to turn off your cell phones. You absolutely do not need to place your cell phones on the front desk.

Recall that it is sufficient to solve two out of the three present exam problems. Feel free to hand in your submission at any time and leave the room without disturbing the other participants.

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[Gravity] Gravity here acts in negative $z$ direction, so that $g_{x}=g_{y}=0$ and $g_{z}=-g=-9.807 \mathrm{~m} \mathrm{~s}^{-2}$.

## [Properties of liquid water]

In all problems on liquid water, the influence of the pressure $p$ on the density $\rho$, the kinematic viscosity $v$, and the surface tension $\gamma$ can be neglected. These properties can here be assumed to depend on the temperature $T$ only, for which correlation expressions

$$
\rho(T)=a T^{2}+b T+\rho_{0}, \quad v(T)=c T+v_{0}, \quad \gamma(T)=\gamma_{0}-\alpha^{-1} T,
$$

can be used, where $a=-0.00343 \mathrm{~kg} \mathrm{~m}^{-3} \mathrm{~K}^{-2}, b=1.79 \mathrm{~kg} \mathrm{~m}^{-3} \mathrm{~K}^{-1}$, $\rho_{0}=768 \mathrm{~kg} \mathrm{~m}^{-3}, c=-1.6 \cdot 10^{-8} \mathrm{~m}^{2} \mathrm{~s}^{-1} \mathrm{~K}^{-1}, v_{0}=5.8 \cdot 10^{-6} \mathrm{~m}^{2} \mathrm{~s}^{-1}$, $\gamma_{0}=0.12 \mathrm{~kg} \mathrm{~s}^{-2}$, and $\alpha=6200 \mathrm{~kg}^{-1} \mathrm{~s}^{2} \mathrm{~K}$. Recall that $v=\mu / \rho$.
[Problem F1] Liquid water flows horizontally, in positive $x$ direction, through a 1.2 m long perfectly smooth $(e=0)$ cylindrical pipe with a diameter of 0.5 mm . The water is heated while it flows, causing its temperature to increase from 280 K at the inlet linearly to 340 K at the outlet; assume that $T$ depends on $x$ only, with a constant value of $d T / d x$. The flow is steady, and the mass flow rate is $0.2 \mathrm{~g} \mathrm{~s}^{-1}$.
Determine the total viscous pressure drop in the pipe, i.e., the decrease from the inlet pressure to the outlet pressure due to head loss. In a perfectly smooth cylindrical pipe, $f_{D}=64 / \mathrm{Re}$ for laminar flow, and $f_{D}=0.316 \operatorname{Re}^{-1 / 4}$ for turbulent flow with $\operatorname{Re}_{c}<\operatorname{Re}<10^{5}$ (assume $\operatorname{Re}_{c}=2320$ ).
[Problem F2] A low-density self-propelling device, which has a flat shape, is oriented horizontally (i.e., perpendicular to the $z$ axis), has a vertical elongation of 0.5 mm , and advances at a constant speed of $u=u_{x}=0.4 \mathrm{~m} \mathrm{~s}^{-1}$, is driven by a motor that continuously supplies technical power; assume that the motor has $100 \%$ efficiency. Between the device and the ground surface (which is also flat and oriented horizontally), there is a layer of liquid water with 2.4 mm thickness. The ground does not move. The force exerted by the motor exactly compensates the viscous shear force. If all involved bodies, in particular the liquid water, are at a temperature of 333 K , the motor has to provide a power of $1.56 \cdot 10^{-4} \mathrm{~W}$.
The temperature decreases to 288 K . The constant speed of the device, the liquid layer thickness, the shape of the device and the ground surface (both horizontal and perfectly flat), and the motor efficiency remain the same. How much power does the motor have to provide at $\mathbf{2 8 8} \mathrm{K}$ ?
[Problem F3] A cylindrical capillary (which, at its top side, is open to the atmospheric air) is submerged vertically into liquid water (in contact with the same atmospheric air), initially at 290 K , which - at that temperature - rises in the capillary to a level 45 mm higher than the level of the air-water interface outside the capillary. At 290 K, water has a contact angle $\vartheta=45^{\circ}$ with the capillary wall material.
The temperature increases to 350 K , while the pressure of the air, 101.3 kPa , is constant. The contact angle of water with the capillary wall material is $\vartheta=35^{\circ}$ at a temperature of 350 K .
Give a) the capillary radius and b) the level to which the water rises in the capillary at 350 K .
The pressure difference at a spherically curved gas-liquid interface is $\Delta p=2 \gamma / r$, where $r$ is the radius of curvature, which in a cylindrical capillary is related by $R=r \cos \vartheta$ to the capillary radius $R$.

Bonus problem:
The Bernoulli equation applies to inviscid flow of an incompressible liquid. What analogous equation can be deduced for the inviscid flow of an ideal gas? Give a deduction, and state in which way your equation is analogous to the Bernoulli equation. Recall that for an ideal gas, the specific enthalpy $h(T)$ is a function of temperature only, with $d h / d T=c_{p}$, where $c_{p}$ is the specific isobaric heat capacity which here for a given ideal gas may be assumed to be constant.

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