## ENGR 356 – FLUID MECHANICS – SECTION 1 – SPRING 2018

## QUIZ II

Please consider the following information.

- You have **75 minutes** from the moment that the beginning of the quiz is announced.
- This quiz consists of a single problem which has five parts. Each part is worth up to three credits, out of 100 credits for the course, but only **up to nine credits** can be gained from this quiz. The full grade is already reached if three of the five parts are solved correctly.
- While in quiz I, up to six credits could be reached, yielding up to 15 available credits from quizzes in principle, the total from quizzes is capped at a maximum of ten credits.

Partial results give partial credits – do what you can, even if your solution is incomplete.

If you need less than 75 minutes, feel free to hand in your results at any time and leave the room without disturbing the other participants. Any discussion with other students or access to means of communication is a case of cheating irrespective of what is communicated.

Address your concerns of any type to the ENGR vice head of department Raguez Taha.

***In your own interest, please avoid unnecessarily long answers.***

**AUIS student ID number: ______________________**

**Full name: ____________________________________**

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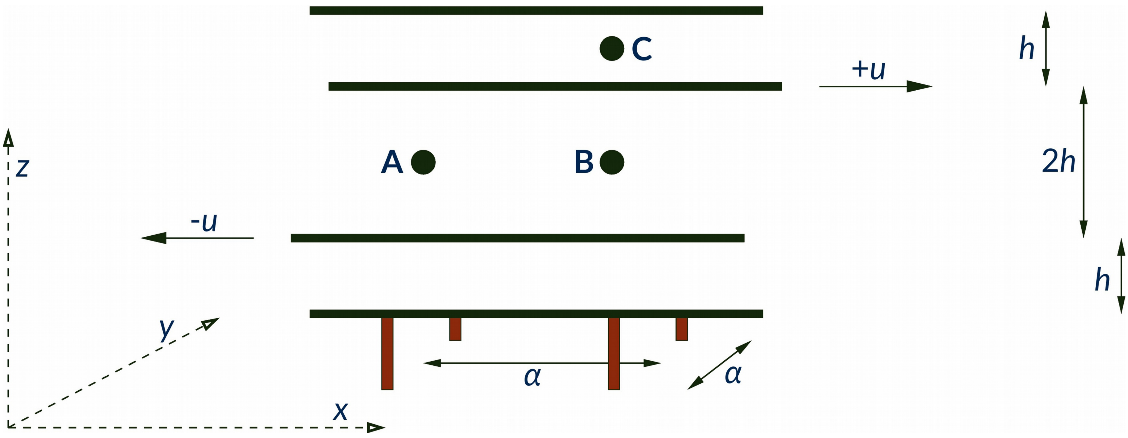


**Liquid water** undergoes a **laminar steady shear flow** between four **planar** and parallel plates, creating one slit-like (planar geometry) channel with a height $2h$ = 1.2 cm, surrounded by two slit-like channels with a height $h$ = 6 mm as sketched above. From bottom to top, the lowest plate is at rest ($u_x = 0$), the second plate moves leftward with $u_x = -u = -0.12$ m s$^{-1}$, the third plate moves rightward with $u_x = +u = +0.12$ m s$^{-1}$, and the plate on top is at rest ($u_x = 0$).

The plates are normal to the $z$ axis and infinitely large in $x$ and $y$ directions. In $y$ and $z$ direction, they all do not move. The no-slip boundary condition can be employed, i.e., fluid in immediate contact with a plate can be assumed to have the same velocity as the plate. Inside the planar channels, there is liquid water everywhere. The bottom plate rests on a square grid of pillars, regularly arranged with a distance $\alpha$ = 1.5 cm between two adjacent pillars.

The bullets do not represent submerged objects – they represent positions. The $z$ coordinates for points A, B, and C are in the middle of the respective channels. At point A, the pressure is $p_A$ = 97.24 kPa. Neglect the mass of the plates, assume that $\partial p/\partial x = \partial p/\partial y = 0$ at all points, assume water to be **incompressible**, with $\rho$ = 999.1 kg m$^{-3}$ and $\mu$ = 1.125 mPa s (both constant), and $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = -p$ here. Gravity **g** is present, with $g_z = -9.807$ m s$^{-2}$, $g_x = g_y = 0$.

You can use the result that the flow under these conditions is unidirectional, i.e., that the velocity field $\mathbf{u}(x, y, z, t)$ reduces to a function $u_x(z)$, while $u_y = u_z = 0$ everywhere at all times.

**Part 1:** Simplify the Navier-Stokes equations (incompressible) as far as possible, starting at

$$\rho \frac{D}{Dt} u_x = \rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right)\right],$$

$$\rho \frac{D}{Dt} u_y = \rho g_y + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)\right] + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right)\right],$$

$$\rho \frac{D}{Dt} u_z = \rho g_z + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)\right] + \frac{\partial \sigma_{zz}}{\partial z}.$$

**Part 2:** Prove that, within each slit, $u_x(z)$ is linear, i.e., that $d^2u/dz^2 = 0$ everywhere.

**Part 3:** Determine the stress tensor $\sigma$ at point B and the stress tensor at point C.

**Part 4:** Determine the force in $x$ and $z$ direction that each pillar exerts on the bottom plate.

**Part 5:** What scale, or unit, of mass $[m]$, what length scale $[l]$, and what time scale $[t]$ need to be chosen to obtain reduced values of unity for the (reduced) characteristic slit diameter $h^* = h/[l] = 1$, the (reduced) characteristic plate velocity $u^* = u/[l\,t^{-1}]$, and the (reduced) specific weight $\rho^* g^* = \rho g/[m\,l^{-2}t^{-2}] = 1$?

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**QUIZ II**

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