ENGR 356 – FLUID MECHANICS – SECTION 1 – SPRING 2018 – TERM EXAM II

Recommendation: Solve two out of the three given problems.

- You have **110 minutes** from the moment that the beginning of the exam is announced.
- This exam consists of three problems. Each is worth up to ten credits, out of 100 credits for the whole course. At most 15 credits can be gained from the present term exam.

  You need to **work on two problems** to achieve an optimal outcome.
- If you **choose to work on two problems**, the problem with the best outcome will count normally (i.e., up to ten credits), and the other problem will be scaled by a factor of 50% (i.e., up to five credits), yielding an **optimum total of 15 credits**.
- If you choose to work on all three problems, the outcomes will be ordered by the number of credits achieved. **The best problem counts with a factor of 100%**, and **the second best problem is scaled by a factor of 50%**, yielding up to 15 credits for the exam as a whole. The outcome of the remaining problem does not influence your grade.
- On the final page, there is also a bonus problem, yielding up to five credits additionally.

Make sure that **every paper** that you submit contains your **name** and **student ID**. Any access to means of communication is a case of cheating irrespective of what is communicated. It is enough to **turn off your cell phones**. *You absolutely do not need to place your cell phones on the front desk.*

Recall that it is sufficient to **solve two out of the three present exam problems**. Feel free to hand in your submission at any time and leave the room without disturbing the other participants.

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[Problem II.1: Poiseuille flow]

Supercritical carbon dioxide flows downward through a vertically oriented cylindrical pipe; i.e., the fluid flow occurs in negative z direction, and the pipe orientation is parallel to the z axis, where z is the direction in which gravity acts, with $g_z = -g = -9.807$ m s^{-2}, whereas $g_x = g_y = 0$. Supercritical fluid carbon dioxide is not an incompressible liquid, nor is it an ideal gas.

The pressure $p = 15.0$ bar, density $\rho = 25.0$ kg m^{-3}, and temperature $T = 61.84$ °C of the carbon dioxide are all the same at the pipe inlet and the pipe outlet; assume the flow to be steady, uni-directional, compressible, viscous, and laminar. The diameter of the cylindrical pipe D is constant throughout (cylindrical shape), the pipe wall material is perfectly smooth (zero roughness, $e = 0$), and the dynamic viscosity $\mu = \rho\nu = 1.684 \cdot 10^{-5}$ Pa s is here also constant.

a) Simplify the Navier-Stokes equation for the z component as far as possible,[1] beginning with

$$\rho \frac{D}{Dt} u_z = \rho g_z + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)\right] + \frac{\partial \sigma_{zz}}{\partial z}.$$

Note that the flow is steady and the conditions described above imply that the pressure, the temperature, the density, the viscosity, and the channel diameter are all invariant in z direction.

b) The Darcy friction factor for vertical Poiseuille flow is $f_D = 2g\hat{u}^{-2}D$, where $\hat{u}$ is the average flow velocity, whereas for horizontal pipes it was defined as $f_D = |dp/dx| \cdot 2\rho^{-1}\hat{u}^{-2}D$ in the lecture. Can you explain why g now replaces $|dp/dx|\,\rho^{-1}$? In the present case, why is the pressure gradient zero even though there is viscous friction? Give a **concise** statement[2] as an explanation.

c) Determine the smallest friction factor f_D compatible with the conditions above (in particular, with laminar flow, assuming that this breaks down at $\mathrm{Re}_c = 2320$), if $f_D = 64/\mathrm{Re}$ for laminar flow.

What is the range of values for the pipe diameter D with which, at the given conditions, it is reasonable to assume laminar flow, i.e., the critical Reynolds number $\mathrm{Re}_c = 2320$ is not exceeded?

PLEASE DO NOT REPEAT "KNOWNS" AND "UNKNOWNS" FROM THE TEXT UNLESS NECESSARY.

Remain succinct – useless or overlong text does not improve the exam result, it only costs time.

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1 *You are **not** expected here to transform the equation to cylindrical coordinates or to determine the $u_z(x, y)$ field.*
2 *Strict limit: 60 words; an equation counts as ten words. If your explanation is longer, it will not be considered.*

[Problem II.2: Couette flow]

A cylindrical solid plug with the mass m_p = 11.79 g, the diameter D_p = 15 mm, and the elongation Δz_p = 9.0 mm descends (in negative z direction) in the middle of a thin cylindrical channel which contains liquid water and has the diameter D_c = 15.1 mm and the length L_c = 11.3 m. The vertical velocity of the plug $u_z < 0$ is constant as it keeps descending through the narrow channel.

The symmetry axis of the cylindrical plug and the cylindrical channel is the same; the plug descends through the channel in a middle position. The symmetry axis is aligned vertically, i.e., parallel to the z axis. The channel itself does not move, i.e., its walls are at rest, and it is surrounded by water; it is open both at its top and its bottom end, and both openings are connected to the same infinite reservoir of liquid water. Gravity acts in z direction, $g_z = -g = -9.807$ m s^{-2}, while $g_x = g_y = 0$. At the inlet of the channel, i.e., at its top opening, the gage pressure is +5 kPa.

The density of the (approximately incompressible) water is ρ = 999.7 kg m^{-3}, and its kinematic viscosity is $\nu = \mu/\rho = 1.345 \cdot 10^{-6}$ m^2 s^{-1}. The shear flow between the plug and the channel wall is viscous and laminar. Viscous forces need to be taken into account for the shear flow between the plug and the channel wall only. The flow in all other regions can be approximated as inviscid.

a) Balancing shear forces, buoyancy, and gravity, what value is obtained for the constant velocity u_z of the falling plug? Neglect any other forces acting on the plug beside those mentioned here, neglect all curvature effects, and treat this as steady laminar shear flow between parallel plates.

b) Determine the gradient of the stress tensor,[3] defined here as in the lecture by $(\nabla^T\sigma)^T$, for a point on the symmetry axis of the channel where shear stresses can be neglected, 270 mm below the inlet of the channel, but much higher than the top end of the plug which is situated further below. What is the gage pressure at this point? Therein, σ is the stress tensor, a 3 x 3 matrix, to which the (divergence or gradient) operator ∇ is here applied using vector notation.

c) What is the Reynolds number Re = hu/ν, following the usual conventions, for the Couette shear flow between the plug and the channel walls? The symbols in the equation given here may have a different meaning than in other cases; recall that in the definition of the Reynolds number, h is the characteristic length scale and u is the characteristic velocity scale for any given scenario.

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3 *No proof or long deduction and argumentation is needed; it is sufficient to just give the result.*

[Problem II.3: Scaling]

Conduct a dimensionality and scaling analysis for one of the two preceding problems as follows:

a) Choose any of the two problems given above, gravity-driven Poiseuille flow of a compressible fluid at invariant pressure (Problem II.1) <u>**or**</u> Couette shear flow applied to the vertical motion of a solid plug through a narrow channel (Problem II.2). Make it clear which problem you discuss.

Give a list of problem parameters; this should include all quantities that appear in the relevant equation or, if the problem is best described by a system of equations, in any of these equations.

b) Decide which parameters you select[4] to reduce to unity, so that the reduced values become 1.

c) Determine scaling parameters (also “primary dimensions”), i.e., effective units by which all problem parameters can be reduced, e.g., the length scale [l], the mass scale [m], and the time scale [t], as a function of the problem parameters, under the condition that the problem parameters which were accordingly selected in **b)** are all reduced to 1.

If there are more problem parameters than scaling parameters (i.e., the number of problem parameters n is greater than the applicable number of primary dimensions m), introduce an appropriate number $n - m$ of additional dimensionless parameters Π_i to characterize how a problem instance can still be varied even after the parameters selected in **b)** have all been reduced to 1.

d) For the concrete problem instance described in the problem statement of Problem II.1 or Problem II.2, respectively, determine concrete values of all parameters introduced in **c)**, i.e., of the scaling parameters (primary dimensions, e.g., [l], [m], [t]) and the dimensionless parameters Π_i.

[Bonus problem – not needed for the full grade!]

Transform the relevant equation(s) of Problem II.1 <u>**or**</u> of Problem II.2 to a dimensionless version.[5]

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4 *The units of these quantities must be independent algebraically, otherwise this is impossible; e.g., it is impossible to reduce both the cross-sectional area of a cylindrical pipe and its diameter to unity, since the unit of length and the unit of area are not independent – if the diameter is 1, the cross-sectional area automatically becomes $\pi/4$.*

5 *This means that all values are replaced by the respective reduced values, which should be eliminated if they are 1.*

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